The 56th Symposium on Ring Theory

and Representation Theory

ABSTRACT

Tokyo Gakugei University, Tokyo

September 16 – 18, 2024

Program

September 16 (Monday)

9:55–10:00 Opening remarks

10:00–10:30 Ryoichi Kase (Okayama University of Science), Toshitaka Aoki (Kobe University), Osamu Iyama (The University of Tokyo), Akihiro Higashitani (Osaka University), Yuya Mizuno (Osaka Metropolitan University)

g-fans of rank 2

10:45–11:15 Shuji Fujino (Tokyo University of Science), Yuta Kozakai(Tokyo University of Science), Kohei Takamura (Tokyo University of Science)

Construction of two-sided tilting complexes for generalized Brauer tree algebras

11:30–12:00 Kyoichi Suzuki (Tokyo University of Science), Naoko Kunugi (Tokyo University of Science)

Lifting of relative stable equivalences of Morita type for blocks of finite groups

13:30–14:00 Yutaka Yoshii (Ibaraki University)

Some commutation fomulas and linear isomorphisms for the hyperalgebra of a simple algebraic group

14:15–15:05 Wahei Hara (Kavli IPMU, The University of Tokyo)

Semibricks and spherical objects

15:30–16:00 Masahide Konishi

On the center of a wreath product of truncated polynomial algebras

16:15–16:45 Cindy Tsang (Ochanomizu University), Yuta Kozakai (Tokyo University of Science)

On skew braces: similarities with rings and groups and their representations

17:00–17:30 Arashi Sakai (Nagoya University), Yuta Kozakai (Tokyo University of Science)

Clifford's theorem in wide subcategories

September 17 (Tuesday)

10:00–10:30 Mayu Tsukamoto (Yamaguchi University), Takahide Adachi (Yamaguchi University), Yuta Kimura (Hiroshima Institute of Technology), Aaron Chan (Nagoya University)

Quasi-hereditary structures and tilting modules

10:45–11:15 Yuta Kimura (Hiroshima Institute of Technology), Osamu Iyama (The University of Tokyo), Kenta Ueyama (Shinshu University)

Tilting for Artin–Schelter Gorenstein algebras of dimension one

11:30–12:00 Riku Fushimi (Nagoya University)

The correspondence between silting objects and t-structures for non-positive dg algebras

13:30–14:00 Satoshi Usui (Tokyo Metropolitan College of Industrial Technology), Takahiro Honma (Yuge College)

The stable category of Gorenstein-projective modules over a monomial algebra

14:15–15:05 Naoya Hiramae (Kyoto University), Yuta Kozakai (Tokyo University of Science)
τ-Tilting finiteness of group algebras and p-hyperfocal subgroups

15:30–16:00 Osamu Iyama (The University of Tokyo), Aaron Chan (Nagoya University), Rene Marczinzik (University of Bonn)
Auslander–Reiten's Cohen–Macaulay algebras and contracted preprojective algebras

16:15–16:45 Kazunori Nakamoto (University of Yamanashi), Takeshi Torii (Okayama University)
The moduli of 5-dimensional subalgebras of the full matrix ring of degree 3

18:00– Conference dinner

September 18 (Wednesday)

10:00–10:30 Ryo Kanda (Osaka Metropolitan University)
Roos categories

10:45–11:15 Maiko Ono (Okayama University of Science), Saeed Nasseh (Georgia Southern University), Yuji Yoshino (Okayama University)
The Atiyah class of a dg module and naïve liftings

11:30–12:00 Yuya Otake (Nagoya University), Kaito Kimura (Nagoya University), Ryo Takahashi (Nagoya University), Justin Lyle (Simons Laufer Mathematical Sciences Institute)
On the existence of counterexamples for vanishing problems of Ext and Tor

13:30–14:00 Ryo Takahashi (Nagoya University)
Finiteness of Orlov spectra of singularity categories

14:15–15:05 Yuta Takashima (Tokyo Metropolitan University), Hokuto Uehara (Tokyo Metropolitan University)
Singularity categories of rational double points in arbitrary characteristic

15:30–16:00 Shunya Saito (The University of Tokyo)
Torsion-free classes of smooth projective curves of genus 0 and 1

16:15–16:45 Masaki Matsuno (Tokyo University of Science), Ayako Itaba (Tokyo University of Science), Yu Saito (Shizuoka University)
Defining relations of 3-dimensional cubic AS-regular algebras whose point schemes are reducible

17:00–17:30 Izuru Mori (Shizuoka University)
Classification of locally free sheaf bimodules of rank 2 over a projective line

g-fans of rank 2

Toshitaka Aoki, Akihiro Higashitani, Osamu Iyama, Ryoichi Kase, and Yuya Mizuno

The notion of tilting objects is central to control equivalences of derived categories. The class of silting objects gives a completion of the class of tilting objects with respect to mutation, which is an operation to replace a direct summand of a given silting object to construct a new silting object. For each finite dimensional algebra A, the collection of (basic) 2-term presilting objects forms a simplicial complex. Furthermore, this gives rise to a nonsingular fan in the real Grothendieck group of A [2], which we call the g-fan of A. g-fans have information about the mutation and also have a property called sign-coherence.

Definition 1. A *sign-coherent fan* is a pair (Σ, σ_+) satisfying the following conditions.

(a) Σ is a nonsingular fan in $\mathbb{R}^n$, and $\sigma_+, -\sigma_+$ are cones of dimension n contained in Σ.
(b) Take $e_1, \ldots, e_n \in \mathbb{R}^n$ such that $\sigma_+ = \mathsf{cone}\{e_i \mid 1 \leq i \leq n\}$. Then for each $\sigma \in \Sigma$, there exists $\epsilon_1, \ldots, \epsilon_n \in \{1, -1\}$ such that $\sigma \subseteq \mathsf{cone}\{\epsilon_1 e_1, \ldots, \epsilon_n e_n\}$.
(c) Each cone of dimension $n-1$ is contained in precisely two cones of dimension n.
(d) Each maximal cone has dimension n.

In this talk, we will focus on complete g-fans of rank 2. We show that complete g-fans of rank 2 are precisely sign-coherent fans of rank 2 with $\sigma_+ = \mathsf{cone}\{[\,1\ 0\,], [\,0\ 1\,]\}$, and we explain methods for constructing algebras that realize such fans. This talk is based on [1].

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Graduate School of Human Development and Environment
Kobe University
3-11 Tsurukabuto, Nada-ku, Kobe 657-8501 JAPAN
Email: toshitaka.aoki@people.kobe-u.ac.jp

Department of Pure and Applied Mathematics
Graduate School of Information Science and Technology
Osaka University
1-5 Yamadaoka, Suita, Osaka 565-0871 JAPAN
Email: higashitani@ist.osaka-u.ac.jp

Graduate School of Mathematical Sciences
University of Tokyo
3-8-1 Komaba Meguro-ku Tokyo 153-8914 JAPAN
Email: iyama@ms.u-tokyo.ac.jp

Department of Information Science and Engineering
Okayama University of Science
1-1 Ridaicho, Kita-ku, Okayama 700-0005 JAPAN
Email: r-kase@ous.ac.jp

Faculty of Liberal Arts, Sciences and Global Education / Graduate School of Science
Osaka Metropolitan University
1-1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531 JAPAN
Email: yuya.mizuno@omu.ac.jp

Construction of two-sided tilting complexes for generalized Brauer tree algebras

Shuji Fujino, Yuta Kozakai, Kohei Takamura

Rickard built tree-to-star tilting complexes for Brauer tree algebras [5]. On the other hand, Membrilio-Hernández formulated tree-to-star tilting complexes for generalized Brauer tree algebras (Brauer graph algebras associated to tree-shaped Brauer graphs) [4].

Rickard and Keller showed that there exists a two-sided tilting complex corresponding to an arbitrary tilting complex for finite dimensional algebras [6, 2]. Constructing such a two-sided tilting complex is difficult for practical calculations. Therefore, Kozakai-Kunugi provided an explicit description of a two-sided tilting complex corresponding to a Rickard's tree-to-star tilting complex for Brauer tree algebras [3]. Since Rickard's complexes for Brauer tree algebras are generalized to Membrilio-Hernández's complexes for generalized Brauer tree algebras, we anticipate that we can construct a two-sided tilting complex corresponding to a Membrilio-Hernández's complex. In this talk, we explicitly construct a two-sided tilting complex corresponding to a Membrilio-Hernández's tree-to-star tilting complex for generalized Brauer tree algebras.

The strategy to construct the two-sided tilting complex is the same as the one used in Kozakai-Kunugi [3], as follows: taking a bimodule inducing a stable equivalence of Morita type corresponding to the Membrilio-Hernández's tree-to-star tilting complex, taking a minimal projective resolution of the bimodule, and deleting some direct summands of each term of the resolution. We rely on the structure of rooted trees and perverse equivalences to prove that the constructed complex is indeed a two-sided tilting complex corresponding to the Membrilio-Hernández's complex. This talk is based on [1].

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Shuji Fujino
Department of Mathematics
Graduate School of Science
Tokyo University of Science
Shinjuku, Tokyo 162-8601 JAPAN
Email: 1124702@ed.tus.ac.jp

Yuta Kozakai
Department of Mathematics
Tokyo University of Science
Shinjuku, Tokyo 162-8601 JAPAN
Email: kozakai@rs.tus.ac.jp

Kohei Takamura
Department of Mathematics
Graduate School of Science
Tokyo University of Science
Shinjuku, Tokyo 162-8601 JAPAN
Email: 1122514@alumni.tus.ac.jp

Lifting of relative stable equivalences of Morita type for blocks of finite groups

Naoko Kunugi and Kyoichi Suzuki

This talk is based on [3] and [4]. Let k be an algebraically closed field of positive characteristic p, and G a finite group. Then the group algebra kG has a unique decomposition into a direct sum of indecomposable algebras. Each summand is called a block of kG. There is a unique block B of kG such that $k_G B \neq 0$, called the principal block of kG, where k_G is the trivial kG-module.

Broué [1] introduced the notion of stable equivalence of Morita type. He also gave a method of constructing these for principal blocks. Linckelmann [5] gave an equivalent condition for stable equivalences of Morita type between indecomposable selfinjective algebras to be in fact Morita equivalences.

Morita equivalences of principal blocks have been constructed by lifting stable equivalences of Morita type using Linckelmann's result (see for example [6, 2]). The stable equivalences of Morita type have also been constructed by Broué's method. However we cannot use Broué's method when finite groups have a common nontrivial central p-subgroup.

On the other hand, Wang-Zhang [7] introduced the notion of relative stable equivalence of Morita type for blocks. This is a generalization of stable equivalence of Morita type. In this talk, we generalize the results of Broué and Linckelmann to relative stable equivalences of Morita type in some cases. This generalization gives a method of constructing Morita equivalences for the principal blocks of finite groups having a common nontrivial central p-subgroup.

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Naoko Kunugi
Department of Mathematics
Tokyo University of Science
Kagurazaka 1-3, Shinjuku-ku, Tokyo, 162-8601, Japan
Email: kunugi@rs.tus.ac.jp

Kyoichi Suzuki
Department of Mathematics
Tokyo University of Science
Kagurazaka 1-3, Shinjuku-ku, Tokyo, 162-8601, Japan
Email: 1119703@alumni.tus.ac.jp

Some commutation formulas and linear isomorphisms for the hyperalgebra of a simple algebraic group

吉井　豊 (Yutaka Yoshii)

Let G be a simply connected and simple algebraic group defined and split over $\mathbb{F}_p$. Let $\mathcal{U}$ be the hyperalgebra over $\mathbb{F}_p$ corresponding to G. The algebra $\mathcal{U}$ has generators $e_\alpha^{(n)}$ induced by the root vectors e_α in the corresponding complex Lie algebra. We first give some commutation formulas of the elements $e_\alpha^{(n)}$. Then, we give some $\mathbb{F}_p$-linear isomorphisms induced by multiplication of the algebra $\mathcal{U}$, using a certain $\mathbb{F}_p$-linear map which splits the Frobenius endomorphism on $\mathcal{U}$.

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茨城大学　基礎自然科学野（教育学部担当）
〒 310-8512　茨城県水戸市文京 2-1-1
College of Education
Ibaraki University
2-1-1 Bunkyo, Mito, Ibaraki 310-8512 JAPAN
Email: yutaka.yoshii.6174@vc.ibaraki.ac.jp

Semibricks and spherical objects

Wahei Hara

The aim of this talk is to share our result that any semibrick complex in the derived category of a silting discrete algebra can be completed to a simple minded collection. Roughly speaking, a silting discrete algebra is a finite dimensional algebra whose derived category is everywhere τ-tilting finite, and one can find a lot of examples of such algebras from both algebraic geometry and representation theory. It also will be explained that, if the geometric setting is relevant, the question about semibrick complexes is connected to the classification of spherical objects, which has a connection to mirror symmetry and McKay correspondence. This is all joint work with Michael Wemyss.

Kavli Institute for the Physics and Mathematics of the Universe (WPI)
University of Tokyo
5-1-5 Kashiwanoha, Kashiwa,
Chiba, 277-8583, JAPAN
Email: wahei.hara@ipmu.jp

On the center of a wreath product of truncated polynomial algebras

Masahide Konishi

In this talk, we study the center of a wreath product algebra of a truncated polynomial algebra. Recall the definition of a wreath product algebra. Let R be a ring with unity and let n be a positive integer. For an R-algebra A and the symmetric group S_n, we define a wreath product algebra $A \wr S_n$ as follows:

- $A \wr S_n = A^{\otimes n} \otimes RS_n$ as a set, where RS_n is a group ring,
- for elements $\mathbf{a} = (a_1 \otimes \cdots \otimes a_n) \otimes \sigma, \mathbf{b} = (b_1 \otimes \cdots \otimes b_n) \otimes \tau$ $(a_i, b_i \in A, 1 \leq i \leq n, \sigma, \tau \in S_n)$ of $A \wr S_n$, define a multiplication $\mathbf{ab}$ as $(a_1 b_{\sigma(1)} \otimes \cdots \otimes a_n b_{\sigma(n)}) \otimes \sigma\tau$.

Let G be a finite group. The conjugacy class in a wreath product group $G \wr S_n$ can be described by types (for example, see [1]). Furthermore, the number of the conjugacy class in $G \wr S_n$ is equal to the number of r-partitions of n where r is the number of the conjugacy class in G.

Let N be a positive integer. Let r_i $(1 \leq i \leq N)$ be positive integers. The aim of this talk is to consider an analogous definition of type for a wreath product algebra $A \wr S_n$ in which A is truncated polynomial algebra $R[x_1, \ldots, x_N]/\langle x_1^{r_1}, \ldots, x_N^{r_N} \rangle$, and describe elements of the center of $A \wr S_n$. Furthermore we show the rank of the center of $A \wr S_n$ is equal to the number of $\prod r_i$-partitions of n.

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Email: masahide.konishi0915@gmail.com

On skew braces: similarities with rings and groups and their representations

Yuta Kozakai and Cindy Tsang

In 2007, as a tool to classify the non-degenerate involutive set-theoretic solutions to the Yang-Baxter equation, Rump [5] introduced an algebraic structure called *brace*. In 2017, in order to classify all of the non-degenerate set-theoretic solutions, Guarnieri and Vendramin [1] generalized brace to *skew brace*.

A *skew brace* is a set $A = (A, \cdot, \circ)$ equipped with two group operations $\cdot$ and $\circ$ such that

$$a \circ (b \cdot c) = (a \circ b) \cdot a^{-1} \cdot (a \circ c)$$

holds for all $a, b, c \in A$, where a^{-1} denotes the inverse of a with respect to the $\cdot$ operation. A *brace* is a skew brace $A = (A, \cdot, \circ)$ for which the group $(A, \cdot)$ is abelian. As we shall explain in the talk, brace was first defined as a generalization of radical ring, and skew brace may be viewed as an extension of group. As one naturally expects, skew brace has a lot of similarities with rings and groups.

Recently, there is a trend to try and extend theories from the study of groups to that of skew braces. For example, the following topics have been explored, and one finds that analogs of certain results from group theory extend to the setting of skew braces.

- Factorizations of skew braces [2, 6].
- Isoclinism of skew braces [3].
- Schur covers of skew braces [4].

In the second half of the talk, we shall discuss skew brace representations based on the definition due to Zhu [7] and Letourmy–Vendramin [4]. In particular, we describe a natural analog of Clifford's theorem for skew brace representations and explain how it follows from the usual Clifford's theorem. We also give some explicit examples of irreducible skew brace representations to illustrate that they are more difficult to classify than irreducible group representations.

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Yuta Kozakai
Department of Mathematics
Tokyo University of Science
1-3 Kagurazaka, Shinjuku-ku, Tokyo, Japan
Email: kozakai@rs.tus.ac.jp

Cindy Tsang
Department of Mathematics
Ochanomizu University
2-1-1 Otsuka, Bunkyo-ku, Tokyo, Japan
Email: tsang.sin.yi@ocha.ac.jp

Clifford's theorem in wide subcategories

Yuta Kozakai and Arashi Sakai

In representation theory of algebras, *wide subcategories* have been investigated in connection with torsion classes, ring epimorphisms [3], semibricks [5] and so on. A wide subcategory is a subcategory of an abelian category which is closed under taking extensions, kernels and cokernels, in other words, an exact abelian subcategory closed under taking extensions.

In representation theory of finite groups, *Clifford's theorem* given in [1] is one of the most important and fundamental results. It states that for any simple module S over a group algebra kG of a finite group G for a field k, the restriction $\mathsf{Res}S$ of S to any normal subgroup N of G is a semisimple kN-module.

In this talk, we give a generalization of Clifford's theorem to a wide subcategory satisfying a certain condition. The restriction functor Res and the induction functor Ind play key roles in the following result.

Theorem 1 ([2])**.** *Let $\mathcal{W}$ be a wide subcategory of $\mathsf{mod}kG$ stable under $k[G/N] \otimes_k -$ and S be a simple object in $\mathcal{W}$. Then $\mathsf{Res}S$ is a semisimple object of a wide subcategory $\mathsf{Ind}^{-1}(\mathcal{W})$ of $\mathsf{mod}kN$. In particular, $\mathsf{Res}S$ is a semibrick.*

Note that the above recovers Clifford's theorem by taking $\mathcal{W} = \mathsf{mod}kG$. Also, if the field k has a characteristic $p > 0$ and G/N is a p-group, then the condition on a wide subcategory is automatically fulfilled. We give an application of the above result to *simple-minded collections* introduced in [4].

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Department of Mathematics
Tokyo University of Science
1-3, Kagurazaka, Shinjuku-ku, Tokyo, 162-8601, JAPAN
Email: kozakai@rs.tus.ac.jp

Graduate School of Mathematics
Nagoya University
Furocho, Chikusa-ku, Nagoya, Aichi, 464-8602, JAPAN
Email: m20019b@math.nagoya-u.ac.jp

Quasi-hereditary structures and tilting modules

Takahide Adachi, Aaron Chan, Yuta Kimura and Mayu Tsukamoto

Cline, Parshall and Scott ([1]) introduced quasi-hereditary algebras in the context of representation theories of complex Lie algebras and algebraic groups. A quasi-hereditary algebra $(A, \lhd)$ consists of an algebra A and a partial order on the set of isomorphism classes of simple A-modules satisfying certain conditions. If $(A, \lhd_1)$ and $(A, \lhd_2)$ are quasi-hereditary algebras with same standard modules, then we say that two partial orders $\lhd_1$ and $\lhd_2$ are equivalent. We call the equivalence classes of this relation quasi-hereditary structures of A.

One of the important properties of quasi-hereditary algebras is the existence of (Miyashita) tilting modules which arise naturally from each quasi-hereditary structure of an algebra, called characteristic tilting modules ([2]). To give a module theoretical interpretation of characteristic tilting modules, we introduce the notion of IS-tilting modules.

Definition 1. Let A be a basic finite dimensional algebra and let $\leq$ be a total order on the set Λ of isomorphism classes of simple A-modules. Let T be a tilting A-module. The pair $(T, \leq)$ is called an *IS-tilting A-module* if there is an indecomposable decomposition $T = \oplus_{x\in\Lambda} T(x)$ satisfying

$$[T(x) : S(x)] = 1 \text{ for all } x \in \Lambda, \text{ and } [T(x) : S(y)] = 0 \text{ whenever } x < y.$$

We provide a relationship between quasi-hereditary structures and IS-tilting modules.

Theorem 2. *Let A be a basic finite dimensional algebra. Then there is a bijection between the set of quasi-hereditary structures on A and the set of basic IS-tilting A-modules. In particular, IS-tilting modules coincide with characteristic tilting modules.*

Next, we explore the class of algebras such that all tilting modules are IS-tilting. An algebra is called a *quadratic linear Nakayama algebra* if the quiver is a linearly oriented quiver of type $\mathbb{A}$ and the relations are generated by quadratic monomials.

Theorem 3. *Let A be a basic finite dimensional algebra. Then all tilting A-modules are IS-tilting if and only if A is a quadratic linear Nakayama algebra.*

Moreover, we describe combinatorial structures of the poset of tilting modules over quadratic linear Nakayama algebras and derive recursive formulas for enumerating tilting modules, that is, quasi-hereditary structures, for quadratic linear Nakayama algebras.

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Faculty of Global and Science Studies, Yamaguchi University
Yoshida, Yamaguchi 753-8541, Japan
Email: tadachi@yamaguchi-u.ac.jp

Graduate school of Mathematics, Nagoya University
Furocho, Chikusaku, Nagoya 464-8602 Japan
Email: aaron.kychan@gmail.com

Department of Mechanical Systems Engineering, Faculty of Engineering
Hiroshima Institute of Technology
Miyake, Saeki-ku Hiroshima 731-5143, Japan
Email: y.kimura.4r@cc.it-hiroshima.ac.jp

Graduate school of Sciences and Technology for Innovation, Yamaguchi University
Yoshida, Yamaguchi 753-8512, Japan
Email: tsukamot@yamaguchi-u.ac.jp

Tilting for Artin-Schelter Gorenstein algebras of dimension one

Osamu Iyama, Yuta Kimura and Kenta Ueyama

One of the main objects of representation theory of a Cohen-Macaulay ring A is the category $\mathsf{CM}A$ of maximal Cohen-Macaulay modules (CM modules for short). By many results on this category, such as the study of Auslander-Reiten sequcence, the structure of the category is gradually becoming clearer. Moreover, if A is Gorenstein, then the situation is much nicer. In fact, the stable category $\underline{\mathsf{CM}}A$ is a triangulated category, and it is equivalent to the singularity category of A by the result of Buchweitz.

Tilting theory is a powerful tool to study triangulated categories. If an algebraic triangulated category $\mathcal{T}$ admits a tilting object T, then we have a triangle equivalence $\mathcal{T} \simeq \mathrm{K}^{\mathrm{b}}(\mathrm{proj}\ \mathrm{End}_{\mathcal{T}}(T))$. Thus by finding a tilting object in the stable category, we can study the category by using derived categories.

To find a tilting object in the stable category of CM modules, we assume that A is $\mathbb{N}$-graded, and we restrict our category to "locally projective on the punctured spectrum" $\mathsf{CM}_0^{\mathbb{Z}}A$ and its stable category $\underline{\mathsf{CM}}_0^{\mathbb{Z}}A$.

In the case where A is an $\mathbb{N}$-graded commutative Gorenstein ring of Krull dimension one, Buchweitz-Iyama-Yamaura [1] proved that the stable category $\underline{\mathsf{CM}}_0^{\mathbb{Z}}A$ has a tilting object if and only if the Gorenstein parameter of A is non-positive or A is regular.

In this talk, we study when the stable category admits a tilting object over an Artin-Schelter Gorenstein algebra (AS Gorenstein for short). AS Gorenstein algebras were introduced as a noncommutative analog of commutative Gorenstein rings from a context of noncommutative algebraic geometry.

The Gorenstein parameter of a commutative local Gorenstein ring is given by a grading of an extension between the simple module and the ring. Similarly, for an AS Gorenstein algebra A, Gorenstein parameters are defined for each simple modules. To state the main result, we consider the average $p^A_{\mathrm{av}} \in \mathbb{Q}$ of Gorenstein parameters.

Theorem 1. *Let $A = \bigoplus_{i \geq 0} A_i$ be a ring-indecomposable AS Gorenstein algebra of dimension one with the average Gorenstein parameter p^A_{av}. Then the triangulated category $\underline{\mathsf{CM}}_0^{\mathbb{Z}}A$ admits a tilting object if and only if $p^A_{\mathrm{av}} \leq 0$ holds or A has finite global dimension.*

One of main examples of AS Gorenstein algebras is Gorenstein orders over $R = k[x]$. In this case, we see that the endomorphism algebra of a tilting object is the incidence algebra of a certain poset. This talk is based on a preprint [2].

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Graduate School of Mathematical Sciences, The University of Tokyo, 3-8-1 Komaba Meguro-ku Tokyo 153-8914, Japan

Email: iyama@ms.u-tokyo.ac.jp

Department of Mechanical Systems Engineering, Faculty of Engineering, Hiroshima Institute of Technology, 2-1-1 Miyake, Saeki-ku Hiroshima 731-5143, Japan

Email: y.kimura.4r@cc.it-hiroshima.ac.jp

Department of Mathematics, Faculty of Science, Shinshu University, 3-1-1 Asahi, Matsumoto, Nagano 390-8621, Japan

Email: ueyama@shinshu-u.ac.jp

THE CORRESPONDENCE BETWEEN SILTING OBJECTS AND t-STRUCTURES FOR NON-POSITIVE DG ALGEBRAS

Riku Fushimi

Silting objects and t-structures each play important roles in representation theory. For a finite dimensional algebra Λ over a filed k, Koenig and Yang [4] proved that there exists a bijective correspondence between silting objects of $\mathsf{K}^b(\mathsf{proj}\Lambda)$ and algebraic t-structures on $\mathsf{D}^b(\mathsf{mod}\Lambda)$. This bijection is called *ST-correspondence* (S stands for silting object and T stands for t-structure). After their works, ST-correspondence was generalized to the case of locally finite homologically smooth non-positive dg algebras [3] and the case of proper non-positive dg algebras [5, 6].

The main result is the following:

Theorem 1. [1] *Let A be a locally finite non-positive dg k-algebra. Then there exists a bijective correspondence between*

(1) *isomorphism classes of basic silting objects of* $\mathsf{per}A$,
(2) *bounded co-t-structures on* $\mathsf{per}A$,
(3) *isomorphism classes of simple-minded collections of* $\mathsf{D}_{\mathsf{fd}}(A)$,
(4) *algebraic t-structures on* $\mathsf{D}_{\mathsf{fd}}(A)$,

where $\mathsf{per}A$ *is the perfect derived category of A, and* $\mathsf{D}_{\mathsf{fd}}(A)$ *is the finite-dimensional derived category of A.*

The above theorem provides a simultaneous generalization of known results. The key ingredient in proving this theorem is the dg Koszul dual introduced in [2].

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Mathematical section in Interdisciplinary
Department of Mathematics
Nagoya University
Furocho, Chikusaku, Nagoya 464-8602, Japan
Email: fushimi.riku.h9@s.mail.nagoya-u.ac.jp

The stable category of Gorenstein-projective modules over a monomial algebra

Takahiro Honma and Satoshi Usui

This talk is based on [2]. Let Λ be a finite dimensional algebra over a field K. The category $\mathrm{Gproj}\,\Lambda$ of Gorenstein-projective Λ-modules is known to be a Frobenius exact category whose projective objects are precisely projective Λ-modules. Hence its stable category $\underline{\mathrm{Gproj}}\,\Lambda$ carries a structure of a triangulated category. Motivated by this, many authors have described the triangulated structure of the stable category $\underline{\mathrm{Gproj}}\,\Lambda$. For example, for monomial algebras Λ, Chen, Shen and Zhou [1] provided a necessary and sufficient condition under which $\underline{\mathrm{Gproj}}\,\Lambda$ is a semisimple triangulated category. It follows from their result that $\underline{\mathrm{Gproj}}\,\Lambda$ is triangle equivalent to the stable module category of a radical square zero self-injective Nakayama algebra when there exists no non-trivial morphism in $\underline{\mathrm{Gproj}}\,\Lambda$. Moreover, Lu and Zhu [3] proved that if Λ is a 1-Iwanaga-Gorenstein monomial algebra, then $\underline{\mathrm{Gproj}}\,\Lambda$ is triangle equivalent to the stable module category of a self-injective Nakayama algebra. We note that for Nakayama algebras Λ, Ringel [4] obtained a similar result to the aforementioned result of Lu and Zhu.

In this talk, we extend the above results to arbitrary monomial algebras Λ. For this, we first study the stable category $\underline{\mathrm{Gproj}}^{\mathbb{Z}}\Lambda$ of graded Gorenstein-projective Λ-modules, where we think of Λ as a positively graded algebra by setting each arrow to be degree one. We then investigate the stable category $\underline{\mathrm{Gproj}}\,\Lambda$ via the G-covering $\underline{\mathrm{Gproj}}^{\mathbb{Z}}\Lambda \to \underline{\mathrm{Gproj}}\,\Lambda$ obtained by Lu and Zhu [3], where G is the cyclic group generated by the defree shift functor (1). Our main result is the following.

Theorem 1 ([2]). *Let Λ be a monomial algebra. Then $\underline{\mathrm{Gproj}}\,\Lambda$ is triangle equivalent to the stable module category of a self-injective Nakayama algebra.*

For a given monomial algebra Λ, we explicitly describe the corresponding self-injective Nakayama algebra.

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Takahiro Honma
Department of General Education
National Institute of Technology (KOSEN), Yuge College
Ochi, Ehime 794-2593 JAPAN
Email: 99cfqqc9@gmail.com

Satoshi Usui
Department of General Education
Tokyo Metropolitan College of Industrial Technology
8-17-1, Minamisenju, Arakawa-ku, Tokyo 116-8523 JAPAN
Email: usui@metro-cit.ac.jp

τ-Tilting finiteness of group algebras and p-hyperfocal subgroups

Naoya Hiramae and Yuta Kozakai

This talk is based on [6]. Let k be an algebraically closed field of positive characteristic p.

τ-Tilting finite algebras introduced by Demonet–Iyama–Jasso [3] have been studied intensively in recent years since they relate to other properties of algebras for certain finiteness: brick finiteness [3], functorially finiteness of all the torsion classes [3], completeness of g-fans [2], and silting discreteness [1]. In the context of modular representation theory of finite groups, Eisele–Janssens–Raedschelders [4] showed that group algebras of tame type are always τ-tilting finite. Given the classical result that the representation type (representation finite, tame, or wild) of a group algebra kG of a finite group G is determined by a p-Sylow subgroup of G (for example, see [5]), it is natural to ask what structures of groups affect τ-tilting finiteness of group algebras.

In this talk, we will explain that p-hyperfocal subgroups control τ-tilting finiteness of group algebras. We give a sufficient condition for group algebras to be τ-tilting finite in terms of p-hyperfocal subgroups.

Definition 1. For a finite group G, we call $P \cap O^p(G)$ a *p-hyperfocal subgroup* of G, where P means a p-Sylow subgroup of G and $O^p(G)$ denotes the smallest normal subgroup of G such that its quotient is a p-group.

Proposition 2 ([6])**.** *Let R be a p-hyperfocal subgroup of a finite group G. Then a group algebra kG is τ-tilting finite if one of the following holds:*

(1) *R is cyclic.*

(2) *$p = 2$ and R is isomorphic to a dihedral, semidihedral, or generalized quaternion group.*

Does the converse of Proposition 2 also hold? As a positive answer to this question, we show that the converse of Proposition 2 is true in the following case:

Theorem 3 ([6])**.** *Let P be an abelian p-group, H an abelian p'-group acting on P, and R the p-hyperfocal subgroup of $G := P \rtimes H$. Then a group algebra kG is τ-tilting finite if and only if one of the following holds:*

(1) *$p = 2$ and R is trivial or isomorphic to the Klein four group $C_2 \times C_2$.*

(2) *$p \geq 3$ and R is cyclic.*

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Naoya Hiramae
Department of Mathematics
Kyoto University
Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto 606-8502 JAPAN
Email: hiramae.naoya.58r@st.kyoto-u.ac.jp

Yuta Kozakai
Department of Mathematics
Tokyo University of Science
1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601 JAPAN
Email: kozakai@rs.tus.ac.jp

Auslander-Reiten's Cohen-Macaulay algebras and contracted preprojective algebras

Osamu Iyama

This is a joint work with Aaron Chan and Rene Marczinzik.

Auslander and Reiten [AR1] called a finite dimensional algebra A over a field k *Cohen-Macaulay* (*CM* for short) if there is an A-bimodule W which gives equivalences

$$- \otimes_A W : \mathcal{P}^{<\infty}(A) \simeq \mathcal{I}^{<\infty}(A) : \mathrm{Hom}_A(W, -),$$

where $\mathcal{P}^{<\infty}(A) := \{X \in \mathsf{mod}A \mid \mathrm{proj.dim}\, X < \infty\}$ and $\mathcal{I}^{<\infty}(A) := \{X \in \mathsf{mod}A \mid \mathrm{inj.dim}\, X < \infty\}$. Such W is called a *dualizing A-module*, and the following equalities are satisfied:

$$\mathrm{fin.dim}\, A = \mathrm{inj.dim}\, W_A = \mathrm{inj.dim}_A W = \mathrm{fin.dim}\, A^{\mathrm{op}}.$$

Dualizing modules are characterized in terms of tilting theory: Recall that an A-module T is called *cotilting* if the k-dual DT is a tilting A^{op}-module. The set $\mathrm{cotilt}\, A$ of additive equivalence classes of cotilting A-modules has a natural partial order given by $T \geq U \Leftrightarrow \mathrm{Ext}^i_A(T, U) = 0$ for all $i \geq 1$.

Proposition 1. [AR1, 1.3] *An A-bimodule W is a dualizing A-module if and only if the following conditions are satisfied.*

- *The A-module W gives a maximal element in* $\mathrm{cotilt}\, A$.
- *The A^{op}-module W gives a maximal element in* $\mathrm{cotilt}\, A^{\mathrm{op}}$.
- *The natural map $A \to \mathrm{End}_A(W)$ is an isomorphism.*

For example, Iwanaga-Gorenstein algebras are precisely CM algebras with $W = A$, and algebras with finitistic dimension zero on both sides are precisely CM algebras with $W = DA$. Moreover, tensor products of CM algebras are again CM. They seem to be all of the known examples of CM algebras.

In this talk, we give the first non-trivial class of CM algebras. For a quiver Q, its double $\overline{Q}$ is defined by adding a new arrow $a^* : j \to i$ for each arrow $a : i \to j$ in Q. The *preprojective algebra* of Q is the factor algebra of the path algebra $k\overline{Q}$ given by

$$\Pi := k\overline{Q}/\langle \sum_{a \in Q_1} (aa^* - a^*a)\rangle.$$

A *contracted preprojective algebra* of Q is the subalgebra $e\Pi e$ of Π, where e is an arbitrary idempotent of Π [IW]. Our first main result is the following.

Theorem 2. *Each contracted preprojective algebra A of Dynkin type is a Cohen-Macaulay algebra. Moreover,* $\mathrm{fin.dim}\, A$ *is either 0 or 2.*

For a CM algebra A with dualizing module W, the category of *Cohen-Macaulay A-modules* is defined as $\mathsf{CM}\, A := {}^{\perp_{>0}}W$. Clearly $\mathsf{CM}\, A \supset \Omega^d(\mathsf{mod}\, A)$ holds for $d := \mathrm{fin.dim}\, A$. Moreover, the equality holds if A is Iwanaga-Gorenstein. Auslander and Reiten posed a question if the converse holds for $d \geq 1$. We show that a family of contracted preprojective algebras gives a negative answer to this question. In fact, if A is a CM algebra that is additionally d-Gorenstein for $d := \mathrm{fin.dim}\, A$ in the sense of [AR2], then $\mathsf{CM}\, A = \Omega^d(\mathsf{mod}\, A)$ always holds. This is an analogue of a well-known equality for isolated singularities.

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Graduate School of Mathematical Sciences
The University of Tokyo
3-8-1 Komaba Meguro-ku Tokyo 153-8914, Japan
Email: iyama@ms.u-tokyo.ac.jp

The moduli of 5-dimensional subalgebras of the full matrix ring of degree 3

Kazunori Nakamoto and Takeshi Torii

Let k be a field. We say that k-subalgebras A and B of $\mathrm{M}_n(k)$ are equivalent (or $A \sim B$) if $P^{-1}AP = B$ for some $P \in \mathrm{GL}_n(k)$. There are 26 equivalence classes of k-subalgebras of $\mathrm{M}_3(k)$ over an algebraically closed field k.

Definition 1. We say that a subsheaf $\mathcal{A}$ of $\mathcal{O}_X$-algebras of $\mathrm{M}_n(\mathcal{O}_X)$ is a *mold* of degree n on a scheme X if $\mathrm{M}_n(\mathcal{O}_X)/\mathcal{A}$ is a locally free sheaf. We denote by $\mathrm{rank}\mathcal{A}$ the rank of $\mathcal{A}$ as a locally free sheaf.

Proposition 2 ([1, Definition and Proposition 1.1], [2, Definition and Proposition 3.5])**.** *The following contravariant functor is representable by a closed subscheme of the Grassmann scheme* $\mathrm{Grass}(d, n^2)$*:*

$$\begin{array}{llcl} \mathrm{Mold}_{n,d} & : & (\mathbf{Sch})^{op} & \rightarrow & (\mathbf{Sets}) \\ & & X & \mapsto & \{ \ \mathcal{A} \mid \ \mathcal{A} \textit{ is a rank } d \textit{ mold of degree } n \textit{ on } X \ \}. \end{array}$$

In this talk, we describe the moduli $\mathrm{Mold}_{3,5}$ of rank 5 molds of degree 3. There are 6 types of 5-dimensional subalgebras of $\mathrm{M}_3(k)$ over an algebraically closed field k:

$$\mathrm{M}_2 \times \mathrm{D}_1 = \left\{ \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix} \right\}, \ \mathrm{S}_{10} = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & e \end{pmatrix} \right\}, \ \mathrm{S}_{11} = \left\{ \begin{pmatrix} a & b & c \\ 0 & e & d \\ 0 & 0 & a \end{pmatrix} \right\},$$
$$\mathrm{S}_{12} = \left\{ \begin{pmatrix} a & b & c \\ 0 & e & d \\ 0 & 0 & e \end{pmatrix} \right\}, \ \mathrm{S}_{13} = \left\{ \begin{pmatrix} * & * & * \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix} \right\}, \ \mathrm{S}_{14} = \left\{ \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ 0 & 0 & * \end{pmatrix} \right\}.$$

Let $\mathcal{A}$ be the universal mold on $\mathrm{Mold}_{3,5}$. For $x \in \mathrm{Mold}_{3,5}$, denote by $\mathcal{A}(x)$ and $k(x)$ the mold corresponding to x and the residue field of x, respectively. For each subalgebra A of M_3, we set

$$\mathrm{Mold}_{3,5}^{A} = \{ x \in \mathrm{Mold}_{3,5} \mid \mathcal{A}(x) \otimes_{k(x)} \overline{k(x)} \sim A \},$$

where $\overline{k(x)}$ is an algebraic closure of $k(x)$.

Theorem 3 ([3])**.** *We have an irreducible decomposition*

$$\mathrm{Mold}_{3,5} = \overline{\mathrm{Mold}_{3,5}^{\mathrm{M}_2 \times \mathrm{D}_1}} \coprod \overline{\mathrm{Mold}_{3,5}^{\mathrm{S}_{13}}} \coprod \overline{\mathrm{Mold}_{3,5}^{\mathrm{S}_{14}}},$$

whose irreducible components are all connected components. The relative dimensions of $\overline{\mathrm{Mold}_{3,5}^{\mathrm{M}_2 \times \mathrm{D}_1}}$*,* $\overline{\mathrm{Mold}_{3,5}^{\mathrm{S}_{13}}}$*, and* $\overline{\mathrm{Mold}_{3,5}^{\mathrm{S}_{14}}}$ *over* $\mathbb{Z}$ *are all* 4*. Moreover,*

$$\overline{\mathrm{Mold}_{3,5}^{\mathrm{M}_2 \times \mathrm{D}_1}} = \mathrm{Mold}_{3,5}^{\mathrm{M}_2 \times \mathrm{D}_1} \cup \mathrm{Mold}_{3,5}^{\mathrm{S}_{11}}, \ \overline{\mathrm{Mold}_{3,5}^{\mathrm{S}_{13}}} = \mathrm{Mold}_{3,5}^{\mathrm{S}_{13}} \cup \mathrm{Mold}_{3,5}^{\mathrm{S}_{12}}, \ \overline{\mathrm{Mold}_{3,5}^{\mathrm{S}_{14}}} = \mathrm{Mold}_{3,5}^{\mathrm{S}_{14}} \cup \mathrm{Mold}_{3,5}^{\mathrm{S}_{10}}.$$

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Center for Medical Education and Sciences
Faculty of Medicine
University of Yamanashi
Chuo, Yamanashi 409-3898, JAPAN
Email: nakamoto@yamanashi.ac.jp

Department of Mathematics
Okayama University
Okayama 700-8530, JAPAN
Email: torii@math.okayama-u.ac.jp

Roos categories

Ryo Kanda

This talk is based on [Kan24].

In [Roo65], Jan-Erik Roos proved the following result:

Theorem 1 ([Roo65])**.** *Let $\mathcal{C}$ be a Grothendieck category. Then the following are equivalent:*

(1) *$\mathcal{C}$ satisfies Grothendieck's conditions Ab6 and Ab4*.*
(2) *There is a pair consisting of a ring Λ and an idempotent ideal I of Λ such that $\mathcal{C}$ is the quotient category of $\operatorname{Mod}\Lambda$ by the bilocalizing subcategory $\operatorname{Mod}(\Lambda/I)$.*

We refer to a Grothendieck category that satisfies Ab6 and Ab4* as a *Roos category.* The definitions of the conditions Ab6 and Ab4* are as follows:

Definition 2. Let $\mathcal{C}$ be an abelian category that admits direct sums and direct products.

(1) $\mathcal{C}$ is said to satisfy *Ab6* if the following condition is fulfilled: Let M be an object in $\mathcal{C}$ and let $\{\{L_i^{(j)}\}_{i\in I_j}\}_{j\in J}$ be a small family of (upward) filtered sets of subobjects of M. Then
$$\bigcap_{j\in J}\left(\bigcup_{i\in I_j} L_i^{(j)}\right) = \bigcup_{\{i(j)\}_j\in\prod_{j\in J} I_j}\left(\bigcap_{j\in J} L_{i(j)}^{(j)}\right),$$
where the symbol $\bigcup$ means the filtered union of subobjects (that is, the sum of subobjects that are filtered).
(2) $\mathcal{C}$ is said to satisfy *Ab4** if direct products are exact, that is, for every small family of short exact sequences, its termwise direct product is again a short exact sequence.

In [BCJF15], Brandenburg, Chirvasitu, and Johnson-Freyd studied the reflexivity and dualizability of locally presentable linear categories (over a field). They conjectured that every dualizable locally presentable linear category is strongly generated by compact projective objects. It turned out that some Roos categories are counterexamples to the conjecture ([Ste23, Example 3.1.24]), so the conjecture is modified as follows:

Conjecture 3. *Every dualizable locally presentable linear category is a Roos category.*

Stefanich [Ste23, Corollary 3.1.19] proved that every dualizable locally presentable linear category is a Grothendieck category satisfying Ab4*. We observe that it also satisfies Ab6, which concludes the following, confirming Conjecture 3:

Theorem 4. *Let R be a commutative ring, and let $\mathcal{C}$ be an R-linear locally presentable category. Then the following are equivalent:*

(1) *$\mathcal{C}$ is dualizable.*
(2) *$\mathcal{C}$ is a Roos category.*

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Department of Mathematics
Graduate School of Science
Osaka Metropolitan University
3-3-138, Sugimoto, Sumiyoshi, Osaka, 558-8585, Japan
Email: ryo.kanda.math@gmail.com

The Atiyah class of a dg module and naïve liftings

Maiko Ono, Saeed Nasseh, and Yuji Yoshino

Throughout the abstract, R is a commutative ring. Let $\varphi : A \to B$ be a homomorphism of DG R-algebras, where B is free as an underlying graded A-module, and J be the diagonal ideal of φ. Let $B^e = B \otimes_A B$ be an enveloping DG algebra of B and $N = (N, \partial^N)$ be a semi-free right DG B-module. The DG module N is said to be *naïvely liftable to A* if the DG B-homomorphism $\pi_N : N \otimes_A B \to N$ defined by $\pi_N(n \otimes b) = nb$ splits. The notion of naïve lifting of DG modules was introduced by the speakers for the purpose of studying the Auslander-Reiten conjecture.

Denote by δ the universal derivation $B \to J$, that is $\delta(b) = b \otimes 1 - 1 \otimes b$ for $b \in B$. A homogeneous A-homomorphism $\nabla : N \to N \otimes_B J$ is called a connection on N if $\nabla(xb) = \nabla(x)b + x \otimes \delta(b)$ holds for $x \in N$ and $b \in B$. The notion of connection was originally defined by A. Connes in non-commutative geometory. For a connection ∇ on N, write $\alpha(\nabla) := \partial^{N \otimes_B J} \circ \nabla - (-1)^n \nabla \circ \partial^N$. Since $\alpha(\nabla) : N \to N \otimes J(-1)$ is a DG B-homomorphism, $\alpha(\nabla)$ defines the homology class $[\alpha(\nabla)]$ in $\mathrm{Ext}^1_B(N, N \otimes_B J)$. We see that $[\alpha(\nabla_1)] = [\alpha(\nabla_2)]$ holds for any connections ∇_1 and ∇_2 on N. The class $[\alpha(\nabla)]$ is called *the Atiyah class of N*. We give a characterization of the naïve liftability of N in terms of the Atiyah class.

Theorem 1. [1] *Let N be a semi-free right DG B-module. The followings are equivalent.*

(1) *N is naively liftable to A.*
(2) *$[\alpha(\nabla)] = 0$ holds for some connection ∇ on N.*

To analyze the Atiyah class of N, we have extended the notion of connection as follows.

Let N be a semi-free right DG B-module and L be a right DG B^e-module. Let ${}^*\mathrm{Der}_A(B, L) := \bigoplus_{n \in \mathbb{Z}} \mathrm{Der}_A(B, L)_n$ be a set of graded A-derivations from B to L. Then ${}^*\mathrm{Der}_A(B, L)$ has a right DG B-module structure with the differential mapping $\partial^{{}^*\mathrm{Der}_A(B,L)}$ which is defined by $\partial^{{}^*\mathrm{Der}_A(B,L)}(D) = \partial^L \circ D - (-1)^n D \circ d^B$ for $D \in \mathrm{Der}_A(B, L)_n$.

Let $D \in \mathrm{Der}_A(B, L)_n$ be a homogeneous A-derivation of degree n. A homogeneous A-homomorphism $\nabla : N \to N \otimes_B L(n)$ is called a *D-connection* of degree n if $\nabla(xb) = \nabla(x)b + (-1)^{|x|n} x \otimes D(b)$ holds for homogeneous elements $x \in N$ and $b \in B$. We define ${}^*\mathrm{Conn}(N, N \otimes_B L) := \bigoplus_{n \in \mathbb{Z}} \mathrm{Conn}(N, N \otimes_B L)_n$ where $\mathrm{Conn}(N, N \otimes_B L)_n = \{\nabla : N \to N \otimes_B L(n) \mid \nabla \text{ is a } D\text{-connection for some } D \in \mathrm{Der}_A(B, L)_n\}$. Note that ${}^*\mathrm{Conn}(N, N \otimes_B L)$ is a right DG B-module. The next theorem is a main result in our talk.

Theorem 2. [2] *Let N be a semi-free right DG B-module. The following assertions hold.*

(1) *Let L be a right DG B^e-module. Then there is the exact sequence*

$$0 \to {}^*\mathrm{Hom}_B(N, N \otimes_B L) \to {}^*\mathrm{Conn}(N, N \otimes_B L) \xrightarrow{\nu_L} {}^*\mathrm{Der}_A(B, L) \to 0$$

of DG B-modules, where the mapping ν_L is defined by $\nu_L(\nabla) = D$ for a D-connection ∇.

(2) *Let $p : N \otimes_B J \to N \otimes_B J/J^2$ be a natural DG B-homomorphism. The following conditions are equivalent.*
   (a) *For some (δ-)connection $\nabla \in \mathrm{Conn}(N, N \otimes_B J)_0$, the DG B-homomorphism $p \circ \alpha(\nabla) : N \to N \otimes_B J/J^2(-1)$ is null homotopic, that is, $[p \circ \alpha(\nabla)] = 0$ in $\mathrm{Ext}^1_B(N, N \otimes_B J/J^2)$.*
   (b) *The DG B-homomorphism $\nu_{J/J^2} : {}^*\mathrm{Conn}(N, N \otimes_B J/J^2) \to {}^*\mathrm{Der}_A(B, J/J^2)$ splits.*

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(Maiko Ono) Institute for the Advancement of Higher Education, Okayama University of Science, Okayama, 700-0005, JAPAN. *Email*: ono@ous.ac.jp

(Saeed Nasseh) Department of Mathematical Sciences, Georgia Southern University, Statesboro, GA 30460, U.S.A. *Email*: snasseh@georgiasouthern.edu

(Yuji Yoshino) Graduate School of Environmental, Life, Natural Science and Technology, Okayama University, Okayama, 700-8530, JAPAN.
Email: yoshino@math.okayama-u.ac.jp

On the existence of counterexamples for vanishing problems of Ext and Tor

Kaito Kimura, Justin Lyle, Yuya Otake and Ryo Takahashi

This talk is based on [5]. Let R be a commutative noetherian local ring. In commutative algebra, it is a classical subject to study the behavior of tensor products of modules. We say that R satisfies *Auslander's depth condition* [2] if all finitely generated R-modules M and N with $\operatorname{Tor}_i^R(M,N)=0$ for all $i>0$ satisfy the *depth formula*, i.e.,

$$\operatorname{depth}(M\otimes_R N)=\operatorname{depth}M+\operatorname{depth}N-\operatorname{depth}R.$$

Huneke and Wiegand [6] proved that every local complete intersection satisfies Auslander's depth condition, and it was extended to *AB rings* by Christensen and Jorgensen [4]. It has been an open question for several decades now whether every local ring, or even every Gorenstein local ring, satisfies Auslander's depth condition, and much work has been put towards providing sufficient conditions for the depth condition to hold; see [1, 2, 3, 4, 6, 8] for but a few examples. In this talk, we consider the relationship between Auslander's depth condition and weak Gorensteinness in the sense of Ringel and Zhang [7], and the converse of the result of Christensen and Jorgensen. As an application, we prove that there exist a Gorenstein equicharacteristic local unique factorization domain having an isolated singularity which does not satisfy Auslander's depth condition, and a Cohen-Macaulay equicharacteristic local unique factorization domain having an isolated singularity which is not weakly Gorenstein.

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Graduate School of Mathematics
Nagoya University
Furocho, Chikusaku, Nagoya 464-8602, Japan
Email: m21018b@math.nagoya-u.ac.jp
Simons Laufer Mathematical Sciences Institute
17 Gauss Way, Berkeley, California 94720-5070, United States
Email: jlyle106@gmail.com
Graduate School of Mathematics
Nagoya University
Furocho, Chikusaku, Nagoya 464-8602, Japan
Email: m21012v@math.nagoya-u.ac.jp
Graduate School of Mathematics
Nagoya University
Furocho, Chikusaku, Nagoya 464-8602, Japan
Email: takahashi@math.nagoya-u.ac.jp

Finiteness of Orlov spectra of singularity categories

Ryo Takahashi

Throughout, we assume all subcategories are nonempty and strictly full. We first recall some notation on generation in a triangulated category introduced by Bondal and Van den Bergh [2] and Rouquier [6].

Definition 1. Let $\mathcal{T}$ be a triangulated category. For a subcategory $\mathcal{X}$ of $\mathcal{T}$, we denote by $\langle \mathcal{X} \rangle$ the smallest subcategory of $\mathcal{T}$ containing $\mathcal{X}$ and closed under finite direct sums, direct summands and shifts. For subcategories $\mathcal{X}, \mathcal{Y}$ of $\mathcal{T}$, we denote by $\mathcal{X} * \mathcal{Y}$ the subcategory of $\mathcal{T}$ consisting of objects $T \in \mathcal{T}$ such that there exists an exact triangle $X \to T \to Y \rightsquigarrow$ in $\mathcal{T}$ with $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$. We set $\mathcal{X} \diamond \mathcal{Y} = \langle \mathcal{X} * \mathcal{Y} \rangle$. For a subcategory $\mathcal{X}$ of $\mathcal{T}$ we set $\langle \mathcal{X} \rangle_0 = 0$, and $\langle \mathcal{X} \rangle_n = \langle \mathcal{X} \rangle_{n-1} \diamond \langle \mathcal{X} \rangle$ for $n \geqslant 1$. When $\mathcal{X}$ consists of an object X (as a strictly full subcategory), we simply write $\langle X \rangle_n$ (with $n \geqslant 0$) instead of $\langle \mathcal{X} \rangle_n$.

The *singularity category* $\mathrm{D_{sg}}(R)$ of a noetherian ring R is defined as the Verdier quotient of the derived category $\mathrm{D^b}(\mathrm{mod}\, R)$ by the homotopy category $\mathrm{K^b}(\mathrm{proj}\, R)$. We introduce an invariant of local rings.

Definition 2. Let R be a (commutative and noetherian) local ring with residue field k. We set

$$\mathrm{dx}(R) = \inf\{n \in \mathbb{N} \mid k \in \langle X \rangle_{n+1} \text{ for every } 0 \neq X \in \mathrm{D_{sg}}(R)\} \in \mathbb{N} \cup \{\infty\}$$

and call this the *dominant index* of R. We say that R is *uniformly dominant* if $\mathrm{dx}(R) < \infty$.

By definition, every uniformly dominant local ring is a dominant local ring in the sense of [9]. In this talk, we shall provide certain sufficient conditions for a given local ring to be uniformly dominant.

Next we recall the definitions of the Orlov spectrum of a triangulated category and related invariants.

Definition 3. Let $\mathcal{T}$ be a triangulated category. The *Orlov spectrum* $\mathrm{OSpec}\,\mathcal{T}$ of $\mathcal{T}$ is defined by

$$\mathrm{OSpec}\,\mathcal{T} = \{\mathrm{gt}_{\mathcal{T}}(G) \mid G \in \mathcal{T} \text{ with } \mathrm{gt}_{\mathcal{T}}(G) < \infty\} \subseteq \mathbb{N},$$

where $\mathrm{gt}_{\mathcal{T}}(G) = \inf\{n \in \mathbb{N} \mid \langle G \rangle_{n+1} = \mathcal{T}\}$ is the *generation time* of G. The *(Rouquier) dimension* $\dim \mathcal{T}$ and *ultimate dimension* $\mathrm{udim}\,\mathcal{T}$ of $\mathcal{T}$ are defined as the infimum and supremum of $\mathrm{OSpec}\,\mathcal{T}$, respectively.

The following theorem is one of the main results of [1].

Theorem 4 (Ballard–Favero–Katzarkov)**.** *Let R be a hypersurface local ring. Suppose that R has an isolated singularity. Then* $\mathrm{udim}\,\mathrm{D_{sg}}(R) < \infty$. *Therefore, the Orlov spectrum* $\mathrm{OSpec}\,\mathrm{D_{sg}}(R)$ *is a finite set.*

In this talk, we confirm that this theorem holds for a uniformly dominant local ring (satisfying certain mild assumptions), and thus, it turns out that for those local rings which satisfy the conditions mentioned above, the singularity category has finite Orlov spectrum, whenever it has an isolated singularity.

If time permits, we give an application to G-dimension, which unifies and refines main results of [7, 8].

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Graduate School of Mathematics
Nagoya University
Furocho, Chikusaku, Nagoya 464-8602, Japan
Email: takahashi@math.nagoya-u.ac.jp

Singularity categories of rational double points in arbitrary characteristic

Yuta Takashima and Hokuto Uehara

This talk is based on [4]. The *singularity category* $\mathbf{D}^{\mathrm{sg}}(X)$ of a quasi-projective variety X is the Verdier quotient of the derived category $\mathbf{D}^{\mathrm{b}}(\mathrm{Coh}\,X)$ by the full subcategory $\mathbf{Perf}(X)$ of perfect complexes. Since $\mathbf{D}^{\mathrm{sg}}(X)$ is trivial if and only if X is non-singular, $\mathbf{D}^{\mathrm{sg}}(X)$ can be thought of what measures complexity of singularities. We are interested in the idempotent-completion $\overline{\mathbf{D}^{\mathrm{sg}}(X)}$, which turns out to be triangulated equivalent to $\mathbf{D}^{\mathrm{sg}}(\widehat{\mathscr{O}}_{X,p})$ if X has only one isolated Gorenstein singular point $p \in X$, rather than $\mathbf{D}^{\mathrm{sg}}(X)$ itself. In fact, quasi-projective varieties X and Y whose formal completions $\widehat{X}$ and $\widehat{Y}$ along singular loci are isomorphic may have non-equivalent singularity categories $\mathbf{D}^{\mathrm{sg}}(X)$ and $\mathbf{D}^{\mathrm{sg}}(Y)$, whereas their idempotent-completions $\overline{\mathbf{D}^{\mathrm{sg}}(X)}$ and $\overline{\mathbf{D}^{\mathrm{sg}}(Y)}$ are triangulated equivalent. Conversely, does a triangulated equivalence $\overline{\mathbf{D}^{\mathrm{sg}}(X)} \simeq \overline{\mathbf{D}^{\mathrm{sg}}(Y)}$ induce an isomorphism $\widehat{X} \cong \widehat{Y}$? The answer is no in general because of Knörrer's periodicity. On the other hand, the answer is yes for the rational double points in characteristic 0 (cf. [1, Proposition 5.8]). The proof depends on their structures of quotient singularities and tautness. Notable facts in comparison with the case in characteristic 0 that the rational double points in positive characteristic are neither quotient singularities nor taut in general lead us to ask what happens in positive characteristic. In this paper, we show the next theorem.

Theorem 1. *Let k be an algebraically closed field and set*

$$\mathsf{Cat} := \{\mathbf{D}^{\mathrm{sg}}(\widehat{\mathscr{O}}_{X,p}) \mid (X,p) \text{ is a rational double point over } k\},$$
$$\mathsf{Dyn} := \{\Delta \mid \Delta \text{ is a simply-laced Dynkin graph}\}.$$

Then a correspondence

$$\begin{array}{ccc} \mathsf{Cat}/(\text{triangulated equivalence}) & \to & \mathsf{Dyn} \\ \mathbf{D}^{\mathrm{sg}}(\widehat{\mathscr{O}}_{X,p}) & \mapsto & \Delta(X,p) \end{array}$$

is a well-defined bijection, where each $\Delta(X,p)$ is the dual graph of the exceptional prime divisors of the minimal resolution of a rational double point (X,p). In particular, if the characteristic of k is 2, 3 or 5, then there exist two rational double points which are not analytically isomorphic but whose singularity categories are triangulated equivalent.

As an application, we construct counter-examples in positive characteristic of the next theorem (1) (and hence (2)), which are the first examples of non-trivial equivalence between singularity categories of hypersurface singularities except Knörrer's periodicity.

Theorem 2 ([2, Theorem 5.9], cf. [3, Theorem 1.4])**.** *Let $R = \mathbb{C}[\![x_0, x_1, \ldots, x_n]\!] / \langle f \rangle$ be an isolated hypersurface singularity.*

(1) *The 0-th Hochschild cohomology of the dg singularity category $\mathbf{D}^{\mathrm{sg}}_{\mathrm{dg}}(R)$ is isomorphic to the Tyurina algebra of f.*
(2) *Let $S = \mathbb{C}[\![x_0, x_1, \ldots, x_n]\!] / \langle g \rangle$ be an isolated hypersurface singularity. If the dg singularity category $\mathbf{D}^{\mathrm{sg}}_{\mathrm{dg}}(S)$ is quasi-equivalent to $\mathbf{D}^{\mathrm{sg}}_{\mathrm{dg}}(R)$, then S is isomorphic to R.*

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Department of Mathematical Sciences
Graduate School of Science
Tokyo Metropolitan University
1-1 Minamiosawa, Hachioji, Tokyo 192-0393, Japan
Email: yuta.takashima.m@gmail.com and hokuto@tmu.ac.jp

Torsion-free classes of smooth projective curves of genus 0 and 1

Shunya Saito

The classification of subcategories of module categories is one of the long-studied topics in the representation theory of algebras. The most classical result is Gabriel's classification of Serre subcategories (= subcategories closed under taking subobjects, quotients, and extensions), which establishes an explicit bijection between the Serre subcategories of the category of coherent sheaves on a noetherian scheme and the specialization-closed subsets of the scheme. In particular, as a special case, he classified the Serre subcategories of the module category of a commutative noetherian ring. Inspired by this result, several classes of subcategories of the module category have been studied and classified so far. On the other hand, there are not many studies on the classification of subcategories of the category of coherent sheaves, even if we focus on the case of smooth projective curves. See [1, 2, 3].

In this talk, I will talk about the classifications of torsion-free classes (= subcategories closed under taking subobjects and extensions) of the category of coherent sheaves on the projective line (genus 0) and an elliptic curve (genus 1).

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Graduate School of Mathematical Sciences
The University of Tokyo
3-8-1 Komaba Meguro-ku Tokyo 153-8914, Japan
Email: shunya.saito712@gmail.com

Defining relations of 3-dimensional cubic AS-regular algebras whose point schemes are reducible

Masaki Matsuno, Yu Saito and Ayako Itaba

In noncommutative algebraic geometry, classification of Artin-Schelter regular (AS-regular) algebras is one of the most important projects. In [1], Artin, Tate and Van den Bergh proved that every 3-dimensional AS-regular algebra finitely generated in degree 1 determines and is determined by a pair (E, σ) where E is a scheme and σ is an automorphism of E. Their work told us that algebraic geometry is a useful tool to study even noncommutative graded algebras.

Let k be an algebraically closed field of characteristic 0 and $A = k\langle x_1, \dots, x_n\rangle/(f_1, \dots, f_m)$ be a cubic algebra where $\deg x_i = 1$ $(i = 1, \dots, n)$ and f_j is a homogeneous element of degree 3 $(j = 1, \dots, m)$. We define Γ_A as follows:

$$\Gamma_A := \{(p, q, r) \in (\mathbb{P}_k^{n-1})^{\times 3} \mid f_j(p, q, r) = 0, \ \ j = 1, \dots, m\}.$$

A pair (E, σ) is called *geometric* if $E \subset (\mathbb{P}_k^{n-1})^{\times 2}$ is a projective variety and σ is an automorphism of E satisfying $\pi_1\sigma = \pi_2$ where $\pi_i : \mathbb{P}_k^{n-1} \times \mathbb{P}_k^{n-1} \to \mathbb{P}_k^{n-1}$ is the i-th projection $(i = 1, 2)$.

Definition 1 ([3], cf.[2])**.** A cubic algebra $A = k\langle x_1, \dots, x_n\rangle/(f_1, \dots, f_m)$ is called *geometric* if there exists a geometric pair (E, σ) such that

(G1) $\Gamma_A = \{(p, q, (\pi_2\sigma)(p, q)) \in (\mathbb{P}_k^{n-1})^{\times 3} \mid (p, q) \in E\}$,

(G2) $(f_1, \dots, f_m)_3 = \{f \in k\langle x_1, \dots, x_n\rangle_3 \mid f(p, q, (\pi_2\sigma)(p, q)) = 0, \ \forall (p, q) \in E\}$.

In this case, we write $A = \mathcal{A}(E, \sigma)$ and E is called the *point scheme* of A.

It is known that every 3-dimensional cubic AS-regular algebra A is a geometric algebra. Moreover, the point scheme E of A is $\mathbb{P}_k^1 \times \mathbb{P}_k^1$ or a curve of bidegree $(2, 2)$ in $\mathbb{P}_k^1 \times \mathbb{P}_k^1$. The following theorem is the main result of this talk.

Theorem 2. *Let $A = \mathcal{A}(E, \sigma)$ be a 3-dimensional cubic AS-regular algebra. Assume that E is either* (i) *a conic and two lines in a triangle,* (ii) *a conic and two lines intersecting in one point, or* (iii) *quadrangle. For each case, we give a complete list of defining relations of A and classify them up to isomorphism and graded Morita equivalence in terms of their defining relations.*

By the results of [3] and Theorem 2, we will give a complete list of defining relations of 3-dimensional cubic AS-regular algebras whose point schemes are reducible.

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Masaki Matsuno and Ayako Itaba
Katsushika division
Institute of arts and sciences
Tokyo University of science
6-3-1 Niijuku, Katsushika-ku, Tokyo, 125-8585, JAPAN
Email: masaki.matsuno@rs.tus.ac.jp
Email: itaba@rs.tus.ac.jp

Yu Saito
Graduate School of Science and Technology
Shizuoka University
836 Ohya, Suruga-ku, Shizuoka-shi, Shizuoka 422-8529 JAPAN
Email: saito.yu.18@shizuoka.ac.jp

Classification of locally free sheaf bimodules of rank 2 over a projective line

Izuru Mori

Classification of noncommutative integral surfaces is one of the fundamental projects since the foundation of noncommuative algebraic geometry. Artin [1] conjectured that every noncommutative integral surface is birationally equivalent to (i) a noncommutative projective plane, (ii) a noncommutative ruled surface, or (iii) a noncommutative surface finite over its center. Since noncommutative projective planes were classified by Artin, Tate and Van den Bergh [2], the next step is to classify noncommutative ruled surfaces. In this talk, we will classify noncommutative Hirzebruch surfaces, which are defined to be noncommutative ruled surfaces over a projective line. To do this, it is essential to classify locally free sheaf bimodules of rank 2 over a projetive line. This talk is based on a joint work with Shinnosuke Okawa and Kazushi Ueda.

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Department of Mathematics
Faculty of Science
Shizuoka University
Ohya 836, Shizuoka 422-8529 JAPAN
Email: mori.izuru@shizuoka.ac.jp