A reduction theorem on Brauer indecomposability of the Scott module

Shigeo Koshitani (Chiba University)

In modular representation theory of finite groups it is quite natural to ask whether two finite dimensional algebras A and B over a field k of a prime characteristic p are derived equivalent, stable equivalent (of Morita type) or even Morita equivalent. Especially if A and B are (so-called) the principal blocks of the group algebras kG and kH of finite groups G and H, respectively, such that G and H have a common Sylow p-subgroup P, in order to check a kind of relations between A and B such as above, an indecomposable (kG, kH)-bimodule $\mathrm{Scott}(G \times H, \Delta P)$ (called the "Scott module") determined by P plays a very important role.

In this talk we are going to discuss a new result on the Brauer indecomposability of the Scott module. This is joint work with Ipek Tuvay.

Derived quiver Heisenberg algebras and Auslander-Reiten theory of the derived category $\mathsf{D}^{\mathrm{b}}(\mathbf{k}Q\,\mathrm{mod})$ of modules of a path algebra

Hiroyuki Minamoto

Let $\mathbf{k}$ be an algebraically closed field of arbitrary characteristic and Q a finite acyclic quiver. An element $v \in \mathbf{k}Q_0$ which we call weight, is said to be sincere if $v_i \neq 0$ for all $i \in Q_0$. For an sincere weight $v \in \mathbf{k}Q_0$, we define a weighted mesh relation ${}^{v}\varrho := \sum_{i \in Q_0} v_i^{-1}\rho_i$ where $\rho_i = \sum_{\alpha: s(\alpha)=i} \alpha\alpha^* - \sum_{\alpha: t(\alpha)=i} \alpha^*\alpha$ denotes the mesh relation at a vertex $i \in Q_0$, so that the usual mesh relation ρ is given as $\rho := \sum_{i \in Q_0} \rho_i$.

Recall that for a sincere weight $v \in \mathbf{k}Q_0$, the derived quiver Heisenberg algebra (DQHA) ${}^{v}\widetilde{\Lambda}(Q)$ is defined as the Ginzberg dg-algebra ${}^{v}\widetilde{\Lambda}(Q) := \mathcal{G}\mathrm{in}(\overline{Q}, W)$ of the double quiver $\overline{Q}$ with the potential

$$W := -\frac{1}{2}{}^{v}\varrho\rho.$$

We need to remark that the above potential W makes sense in the case $\mathrm{char}\,\mathbf{k} \neq 2$ since it has the fraction $\frac{1}{2}$. However in the explicit formula of the differential, the fraction $\frac{1}{2}$ does not appear and hence we can obtain a dg-algebra even in the case $\mathrm{char}\,\mathbf{k} = 2$.

The dg-algebra ${}^{v}\widetilde{\Lambda}(Q)$ acquires extra grading that basically counts the number of the extra arrows α^* for $\alpha \in Q_1$, which we call the $*$-grading of ${}^{v}\widetilde{\Lambda}(Q)$. In my joint work with M. Herschend [1], we established several results that show that the $*$-graded structure of ${}^{v}\widetilde{\Lambda}(Q)$ closely related to Auslander-Reiten theory of the derived category $\mathsf{D}^{\mathrm{b}}(\mathbf{k}Q\,\mathrm{mod})$ of the module category of the path algebra $\mathbf{k}Q$.

In this talk we discuss some details of these results by giving an alternative description of DQHA ${}^{v}\widetilde{\Lambda}(Q)$ which elucidates the above mentioned relationships.

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Connections on dg modules and naïve liftings

Yuji Yoshino

This is a joint work with Maiko Ono (Okayama Universituy of Science) and Saeed Nasseh (Georgia Southern University).

For the past few years, I have been studying, with their collaboration, on the lifting problem of chain complexes over commutative rings. In order to expand the framework so that it can be handled in a unified style, we have reached a new concept called naïvely lifting, whose definition is given as follows:

Let $A \to B$ be a homomorphism of dg algebras over a commutative noetherian ring R. A (semi-free) right dg B-module N is said to be *naïvely liftable to* A if the natural dg B-homomorphism $N|_A \otimes_A B \to N$ splits.

Now let J be the diagonal ideal of $A \to B$. Then there is a short exact sequence of dg B-modules;

$$0 \longrightarrow N \otimes_B J \longrightarrow N|_A \otimes_A B \longrightarrow N \longrightarrow 0.$$

This sequence defines uniquely the class ω in $\mathrm{Ext}^1_B(N, N \otimes_B J)$ which we call the *extended Atiyah class*. Through the natural morphism $J \to J/J^2$, ω is mapped to $\alpha \in \mathrm{Ext}^1_B(N, N \otimes_B J/J^2)$ which, we can prove, is identical to the *classical Atiyah class*. Therefore if N is naïvely liftable to A then the classical Atiyah class of N vanishes. We hope, even conjecture, that under some mild condition on B and N, the vanishing of classical Atiyah class would imply naïve lifting. Maiko Ono is supposed to elaborate on these at the previous meeting.

Recently we have been able to prove the following theorem.

Theorem 1. *Let B be a Tate resolution (or Avramov resolution) of a residue ring R/I of R and N be a semi-free right dg B module that is non-negatively graded. If $\mathrm{Ext}^i_B(N, N) = 0$ for all $i > 0$, then the classical Atiyah class of N vanishes.*

Proof is more difficult than it looks. In general B is a free extension (or polynomial extension) of R with infinite number of variables, hence J, and J/J^2 also, is infinitely generated. This is a difficult point, but we were able to avoid this difficulty by constructing a new theory of generalized connections on dg modules.

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CALABI-YAU COMPLETIONS AND CLUSTER CATEGORIES FROM ROOTS OF DUALIZING BIMODULES

NORIHIRO HANIHARA

Roots of dualizing complexes (or shifted Serre functors) over algebras appear naturally in tilting theory for singularity categories or for projective varieties. Algebraically, they can be formulated as roots of shifted inverse dualizing bimodules over dg categories. As an analogue of Keller's Calabi-Yau completion, we give a construction of Calabi-Yau dg algebras from such roots of shifted inverse dualizing bimodules. We further discuss the cluster category associated to such a Calabi-Yau dg algebra and obvserve that it is an orbit category of the usual cluster category by a finite cyclic group which we call a folded cluster category.

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STABLE MODULE CATEGORIES INSIDE TRIANGULATED CATEGORIES

KIRIKO KATO

The stable category of modules over a noetherian ring is known to be triangulated if the ring is self-injective. In general, the stable category is embedded into a triangulated category as a full subcategory. Thereby we investigate various property of morphisms. In this talk, we introduce recent results on the stable module category from classical viewpoints as well.

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有限テンソル圏における準フロベニウス代数

清水健一

有限群, より一般に有限次元ホップ代数の表現の圏のように, ある種の有限性を持つアーベル圏であって, テンソル積関手 $\otimes$ および双対関手 $(-)^*$ が与えられているようなものを有限テンソル圏 (finite tensor category) と呼ぶ. 有限テンソル圏の最も基本的な例が有限次元ベクトル空間の圏である. 有限次元代数は, この圏の対象であって, いくつかの公理を満足する射が付随しているようなものとして定義することができる. 代数の定義をこのように書き換えてしまうと, それは一般のテンソル圏においても意味を持つようになる. このようにして, 一般のテンソル圏において, 代数や加群の概念を定義することができる. また, すべてではないにせよ, 環論において考えられている様々な種類の代数をテンソル圏において定義することもできる.

テンソル圏におけるフロベニウス代数は, 部分因子環論や共形場理論へのテンソル圏論的なアプローチにおいて重要な役割を持っているため, これまで良く研究されてきた. $\mathcal{C}$ を有限テンソル圏, A を $\mathcal{C}$ における代数とし, $\mathcal{C}_A$ で右 A-加群の圏を表すことにする. このとき $\mathcal{C}$ のテンソル積から誘導される関手 $\mathcal{C} \times \mathcal{C}_A \to \mathcal{C}_A$ があり, これにより $\mathcal{C}_A$ は '左 $\mathcal{C}$-加群圏' となる. $\mathcal{C}$ における代数 A と B は, $\mathcal{C}_A$ と $\mathcal{C}_B$ が左 $\mathcal{C}$-加群圏として同値であるとき, $\mathcal{C}$ において森田同値であるという. テンソル圏の研究および応用においては, テンソル圏におけるフロベニウス代数それ自身よりも, その加群の圏に興味がある場合も少なくない. このことにより, 我々はテンソル圏におけるフロベニウス代数と森田同値な代数の考察へと導かれるのである.

本講演では, 有限テンソル圏における相対セール関手と準フロベニウス代数の理論を概説したい. 上述したような事情を踏まえ, 講演者は, 有限テンソル圏において準フロベニウス代数を定義した. これは通常の環論における準フロベニウス代数を一般化する概念であり, 有限テンソル圏におけるフロベニウス代数やホップ代数を含むクラスの代数でもある. 期待されるように, 有限テンソル圏における代数は, それが準フロベニウスであるとき, かつその時に限り, フロベニウス代数と森田同値となる. このことの証明において重要となるのが, 加群圏の内部 Hom 関手を用いて定義される相対セール関手である. ベクトル空間の圏で考える場合は相対セール関手は中山関手と一致し, 一般の場合は中山関手から '局所コンパクト群上のモジュラー関数の圏論的類似物' の分だけずれていることが示される. 実は準フロベニウス性は相対セール関手の可逆性とも同値である. また, 考えている有限テンソル圏に少々の付加構造を仮定すれば, その圏における対称フロベニウス代数が定義されるが, それもまた相対セール関手によって特徴づけられる.

τ-tilting theory and silting theory of skew group algebra extensions

Yuta Kozakai*

Since the concept of support τ-tilting modules was introduced by Adachi-Iyama-Reiten [1], a lot of classifications and characterizations for them have been given for various kinds of algebras. One reason why we study the modules is that they correspond to many fundamental objects including two-term silting complexes bijectively.

In this talk, we give results which generalize and unify preceding results on support τ-tilting modules and related objects given by Huang-Zhang [3], Breaz-Marcus-Modoi [2], and Koshio-Kozakai [4]. Let Λ be a finite dimensional algebra with an action by a finite group G and $A := \Lambda * G$ the skew group algebra. We explain that the canonical restriction-induction adjoint pair of skew group algebra extension $\Lambda \subset A$ induces a poset isomorphism between the poset of G-stable support τ-tilting modules over Λ and that of $(\operatorname{mod} G)$-stable support τ-tilting modules over A. Moreover we establish that the poset isomorphism between appropriate classes of two-term silting complexes over Λ and A corresponding to the above poset isomorphism extends to those of silting complexes. In addition, as applications of these results, we introduce some results on τ-tilting finiteness and silting discreteness of Λ and so on. This talk is based on joint work with Y. Kimura, R. Koshio, H. Minamoto, and Y. Mizuno [5].

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A survey on the derived categories of module categories over Schur algebras

Hyohe Miyachi (Osaka Metropolitan University)

I’d like to talk about what we have seen in the representation theory of symmetric groups and related algebras in terms of derived equivalences.

M-TF equivalences in the real Grothendieck groups

Sota Asai (The University of Tokyo)

This talk is based on the joint work with Osamu Iyama (arXiv:2404.13232). We consider an abelian category $\mathcal{A}$ of finite length which has only finitely many isoclasses $S_1, \dots, S_n$ of simple objects. Then the real Grothendieck group $K_0(\mathcal{A})_{\mathbb{R}}$ and its dual $\mathbb{R}$-vector space $K_0(\mathcal{A})^*_{\mathbb{R}}$ are identified with the n-dimensional Euclidean space $\mathbb{R}^n$. We mainly focus on the latter $K_0(\mathcal{A})^*_{\mathbb{R}}$. For each element $\theta \in K_0(\mathcal{A})^*_{\mathbb{R}}$, Baumann-Kamnitzer-Tingley defined two semistable torsion pairs $(\overline{\mathcal{T}}_\theta, \mathcal{F}_\theta)$ and $(\mathcal{T}_\theta, \overline{\mathcal{F}}_\theta)$ in $\mathcal{A}$. By using them, I introduced an equivalence relation on $K_0(\mathcal{A})^*_{\mathbb{R}}$ called the TF equivalence in my previous work. The TF equivalence plays an important role in relation to silting theory of finite dimensional algebras. On the other hand, the TF equivalence classes are often very complicated, so to understand them well, we need to coarsen the TF equivalence preserving representation-theoretical meaning. In our study, we introduced the M-TF equivalence for each object $M \in \mathcal{A}$, and showed that the set $\Sigma(M)$ of the closures of all M-TF equivalence classes is the normal fan of the Newton polytope $\mathrm{N}(M)$ in $K_0(\mathcal{A})_{\mathbb{R}}$. I would like to explain such basic properties of M-TF equivalences in this talk.

Tilted algebras behind repetitive algebras

Kunio Yamagata (Tokyo University of Agriculture and Technology)

有限次元多元環の反復多元環に付随する tilted algebras に関する基本事項について概説する.