Lemma 3.1

```
In[1]:=  Solve[{y == t x + s (a x + b y), z == a x + b y}, {y, z}]
```

Out[1]= $\left\{\left\{y \to -\frac{a s x + t x}{-1 + b s},\ z \to -\frac{a x + b t x}{-1 + b s}\right\}\right\}$

```
In[2]:= Solve[(1 - b s) u + (t u + s v) (t + a s) + v (a + b t) == 0, {u}]
```

Out[2]= $\left\{\left\{u \to \frac{a v + a s^2 v + b t v + s t v}{-1 + b s - a s t - t^2}\right\}\right\}$

```
In[3]:= Collect[Expand[
          (x - u)^2 + (t x + s z - t u - s v)^2 + (z - v)^2 - (u^2 + (t u + s v)^2 + v^2)], {z, x}]
```

Out[3]= $\left(-2 u - 2 t^2 u - 2 s t v\right) x + \left(1 + t^2\right) x^2 + \left(-2 s t u - 2 v - 2 s^2 v + 2 s t x\right) z + \left(1 + s^2\right) z^2$

In[4]:= **Solve**$\left[\left(-2 u - 2 t^2 u - 2 s t v\right) x + \left(1 + t^2\right) x^2 + (-2 k + 2 s t x) z + \left(1 + s^2\right) z^2 == 0,\ z\right]$

Out[4]= $\Big\{\Big\{z \to \frac{1}{2\left(1 + s^2\right)}\left(2 k - 2 s t x - \sqrt{\left((-2 k + 2 s t x)^2 - 4\left(1 + s^2\right)\left(-2 u x - 2 t^2 u x - 2 s t v x + x^2 + t^2 x^2\right)\right)}\right)\Big\},$

$\Big\{z \to \frac{1}{2\left(1 + s^2\right)}\left(2 k - 2 s t x + \sqrt{\left((-2 k + 2 s t x)^2 - 4\left(1 + s^2\right)\left(-2 u x - 2 t^2 u x - 2 s t v x + x^2 + t^2 x^2\right)\right)}\right)\Big\}\Big\}$

In[5]:= **Simplify**$\left[s\,(t u + s v) + v\ /.\ u \to \frac{a v + a s^2 v + b t v + s t v}{-1 + b s - a s t - t^2}\right]$

Out[5]= $-\frac{(-1 + b s)\left(1 + s^2 + t^2\right) v}{1 - b s + a s t + t^2}$

Hence, k is $-\frac{(-1 + b s)\left(1 + s^2 + t^2\right) v}{1 - b s + a s t + t^2}$

In[6]:= **Simplify**$\left[1 - (2 k)^{\wedge}(-2)\left((-2 k + 2 s t x)^2 - 4\left(1 + s^2\right)\left(-2 u x - 2 t^2 u x - 2 s t v x + x^2 + t^2 x^2\right)\right)\right]$

Out[6]= $\frac{1}{k^2} x \left(2 k s t - 2\left(1 + s^2\right)\left(1 + t^2\right) u - 2 s t v - 2 s^3 t v + x + s^2 x + t^2 x\right)$

In[7]:= **Series**$\left[-\sqrt{1 - X},\ \{X, 0, 4\}\right]$

Out[7]= $-1 + \frac{X}{2} + \frac{X^2}{8} + \frac{X^3}{16} + \frac{5 X^4}{128} + O[X]^5$

In[8]:= **Factor**$\left[2 k t s - 2 (s^{\wedge}2 + 1)(u + t (t u + s v))\ /.\ \left\{u \to \frac{a v + a s^2 v + b t v + s t v}{-1 + b s - a s t - t^2},\ k \text{->} -\frac{(-1 + b s)\left(1 + s^2 + t^2\right) v}{1 - b s + a s t + t^2}\right\}\right]$

Out[8]= $-\frac{2\left(a + a s^2 + b t + s t\right)\left(1 + s^2 + t^2\right) v}{-1 + b s - a s t - t^2}$

In[9]:= **C0 := $-\frac{2\left(a+a s^2+b t+s t\right)\left(1+s^2+t^2\right) v}{-1+b s-a s t-t^2}$**

In[10]:= **D0 := s^2 + t^2 + 1**

In[11]:= **k := $-\frac{(-1+b s)\left(1+s^2+t^2\right) v}{1-b s+a s t+t^2}$**

A1 is

In[12]:= **Simplify$\left[-(u+t(t u+s v))/k\ /.\ u\to\frac{a v+a s^2 v+b t v+s t v}{-1+b s-a s t-t^2}\right]$**

Out[12]= $\frac{a+b t}{1-b s}$

A2 is

In[13]:= **Simplify[D0 / (2 k (s^2 + 1)) + C0^2 / (8 k^3 (s^2 + 1))]**

Out[13]= $\left(\left(-1+b s-a s t-t^2\right)\left(1-2 b s+a^2\left(1+s^2\right)+2 a(b+s) t+t^2+b^2\left(s^2+t^2\right)\right)\right)/\left(2(-1+b s)^3\left(1+s^2+t^2\right) v\right)$

Here,

In[14]:= **Factor[((s^2 + t^2 + 1) (1 - b s)^2 + (a s^2 + t s + b t + a)^2)]**

Out[14]= $\left(1+s^2\right)\left(1+a^2-2 b s+a^2 s^2+b^2 s^2+2 a b t+2 a s t+t^2+b^2 t^2\right)$

A3 is

In[15]:= **Simplify[2 C0 D0 / (8 k^3 (s^2 + 1)) + C0^3 / (16 k^5 (s^2 + 1))]**

Out[15]= $-\left(\left(a\left(1+s^2\right)+(b+s) t\right)\left(1-b s+a s t+t^2\right)^2\left(1-2 b s+a^2\left(1+s^2\right)+2 a(b+s) t+t^2+b^2\left(s^2+t^2\right)\right)\right)/\left(2(-1+b s)^5\left(1+s^2+t^2\right)^2 v^2\right)$

A4 is

In[16]:= **Simplify[D0^2 / (8 k^3 (s^2 + 1)) + 3 C0^2 D0 / (16 k^5 (s^2 + 1)) + 5 C0^4 / (128 k^7 (s^2 + 1))]**

Out[16]= $\left(\left(1-b s+a s t+t^2\right)^3\left(-5\left(a\left(1+s^2\right)+(b+s) t\right)^4-6(-1+b s)^2\left(a\left(1+s^2\right)+(b+s) t\right)^2\left(1+s^2+t^2\right)-(-1+b s)^4\left(1+s^2+t^2\right)^2\right)\right)/\left(8(-1+b s)^7\left(1+s^2\right)\left(1+s^2+t^2\right)^3 v^3\right)$

In[17]:= **Factor$\left[\left(-5\left(a\left(1+s^2\right)+(b+s) t\right)^4-6(-1+b s)^2\left(a\left(1+s^2\right)+(b+s) t\right)^2\left(1+s^2+t^2\right)-(-1+b s)^4\left(1+s^2+t^2\right)^2\right)\right]$**

Out[17]= $-\left(1+s^2\right)\left(1+a^2-2 b s+a^2 s^2+b^2 s^2+2 a b t+2 a s t+t^2+b^2 t^2\right)\left(1+5 a^2-2 b s+s^2+10 a^2 s^2+b^2 s^2-2 b s^3+5 a^2 s^4+b^2 s^4+10 a b t+10 a s t+10 a b s^2 t+10 a s^3 t+t^2+5 b^2 t^2+8 b s t^2+5 s^2 t^2+b^2 s^2 t^2\right)$

Therefore A4 is

$$\left(1-b\,s+a\,s\,t+t^2\right)^3\left(1+a^2-2\,b\,s+a^2\,s^2+b^2\,s^2+2\,a\,b\,t+2\,a\,s\,t+t^2+b^2\,t^2\right)\left(1+5\,a^2-2\,b\,s+s^2+10\,a^2\,s^2+b^2\,s^2-2\,b\,s^3+5\,a^2\,s^4+b^2\,s^4+10\,a\,b\,t+10\,a\,s\,t+10\,a\,b\,s^2\,t+10\,a\,s^3\,t+t^2+5\,b^2\,t^2+8\,b\,s\,t^2+5\,s^2\,t^2+b^2\,s^2\,t^2\right)\Big/\left(8\,(1-b\,s)^7\left(1+s^2+t^2\right)^3 v^3\right)$$

In[18]:= **Expand[(1 + 5 a^2 − 2 b s + s^2 + 10 a^2 s^2 + b^2 s^2 − 2 b s^3 + 5 a^2 s^4 + b^2 s^4 + 10 a b t + 10 a s t + 10 a b s^2 t + 10 a s^3 t + t^2 + 5 b^2 t^2 + 8 b s t^2 + 5 s^2 t^2 + b^2 s^2 t^2) − ((s^2 + t^2 + 1) (1 − b s)^2 + 5 (a s^2 + t s + b t + a)^2)]**

Out[18]= 0

Proposition 3.3

In[19]:= **A2 := CT / (1 − b s)**

In[20]:= **A3 := (DT + s CT1 A2) / (1 − b s)**

In[21]:= **Expand[A3]**

Out[21]= $\frac{\text{CT CT1 s}}{(1-b\,s)^2}+\frac{\text{DT}}{1-b\,s}$

In[22]:= **A4 := (ET + s CT1 A3 + s DT1 A2 + s^2 c2 A2^2) / (1 − b s)**

In[23]:= **Expand[A4]**

Out[23]= $\frac{\text{c2 CT}^2\,s^2}{(1-b\,s)^3}+\frac{\text{CT CT1}^2\,s^2}{(1-b\,s)^3}+\frac{\text{CT1 DT s}}{(1-b\,s)^2}+\frac{\text{CT DT1 s}}{(1-b\,s)^2}+\frac{\text{ET}}{1-b\,s}$

In[24]:= **Simplify[a s^2 + t s + b t + a /. t → (1 − b s) T − a s]**

Out[24]= $-(-1+b\,s)\,(a+(b+s)\,T)$

In[25]:= **Simplify[(a^2 + b^2) s^2 + 2 (a t − b) s + (b^2 + 1) t^2 + 2 a b t + a^2 + 1 /. t → (1 − b s) T − a s]**

Out[25]= $(-1+b\,s)^2\left(1+a^2+2\,a\,b\,T+\left(1+b^2\right)T^2\right)$

Here R[T] := $1+a^2+2\,a\,b\,T+\left(1+b^2\right)T^2$

In[26]:= **Solve[{(s CT1 CT + (1 − b s) DT) RT == 2 (a + (b + s) T) CT^2}, s]**

Out[26]= $\left\{\left\{s\to\frac{-2\,a\,\text{CT}^2+\text{DT RT}-2\,b\,\text{CT}^2\,T}{-\text{CT CT1 RT}+b\,\text{DT RT}+2\,\text{CT}^2\,T}\right\}\right\}$

this denominator is W := b S + C K because

In[27]:= **Expand[b (DT RT − 2 (b T + a) CT^2) + CT KT /. {KT → (2 (b^2 + 1) T + 2 a b) CT − RT CT1}]**

Out[27]= $-\text{CT CT1 RT}+b\,\text{DT RT}+2\,\text{CT}^2\,T$

In[28]:= **Simplify[(s^2 + t^2 + 1) (1 − b s)^2 + 5 (a s^2 + t s + b t + a)^2 /. t → (1 − b s) T − a s]**

Out[28]= $(-1+b\,s)^2\left(1+s^2+5\,(a+(b+s)\,T)^2+(a\,s+(-1+b\,s)\,T)^2\right)$

```
In[29]:= Collect[Cancel[RT^2 (1 - b s)^3
            (A4 - A2^3 (1 + s^2 + 5 (a + (b + s) T)^2 + (a s + (-1 + b s) T)^2) / RT^2)], s, Simplify]
```

Out[29]= $ET\,RT^2 - CT^3\left(1 + 5a^2 + 10\,a\,b\,T + T^2 + 5b^2T^2\right) + s\left(CT1\,DT\,RT^2 + CT\,DT1\,RT^2 - 2\,b\,ET\,RT^2 - 8\,CT^3\,T\,(a + b\,T)\right) + s^2\left(c2\,CT^2\,RT^2 + CT\left(CT1^2 - b\,DT1\right)RT^2 + b\,(-CT1\,DT + b\,ET)\,RT^2 - CT^3\left(1 + a^2 + 2\,a\,b\,T + \left(5 + b^2\right)T^2\right)\right)$

```
In[30]:= s := ST / WT

In[31]:= Z1 := RT^2 WT^2 (1 - b s)^3
            (A4 - A2^3 (1 + s^2 + 5 (a + (b + s) T)^2 + (a s + (-1 + b s) T)^2) / RT^2)

In[32]:= Collect[Cancel[Z1], {ET, DT, DT1, CT}, Simplify]
```

Out[32]= $c2\,CT^2\,RT^2\,ST^2 + CT\,CT1^2\,RT^2\,ST^2 + CT1\,DT\,RT^2\,ST\,(-b\,ST + WT) + CT\,DT1\,RT^2\,ST\,(-b\,ST + WT) + ET\,RT^2\,(-b\,ST + WT)^2 + CT^3\left(-ST^2\left(1 + a^2 + 2\,a\,b\,T + \left(5 + b^2\right)T^2\right) - 8\,ST\,T\,(a + b\,T)\,WT - \left(1 + 5a^2 + 10\,a\,b\,T + \left(1 + 5b^2\right)T^2\right)WT^2\right)$

As for the coefficient of CT^3,

```
In[33]:= Expand[(-ST^2 (1 + a^2 + 2 a b T + (5 + b^2) T^2) -
            8 ST T (a + b T) WT - (1 + 5 a^2 + 10 a b T + (1 + 5 b^2) T^2) WT^2) +
          (WT^2 + ST^2 + 5 (a WT + (b WT + ST ) T)^2 + (a ST - (WT - b ST) T)^2)]

Out[33]= 0

In[34]:= WT := b ST + CT KT

In[35]:= ST := DT RT - 2 (b T + a) CT^2

In[36]:= ZT := KT^2 (RT ET - CT^3) + RT KT DT (DT1 RT - 3 (b^2 + 1) T DT) + DT^2 RT
            (-a b (2 KT + T KT1) - 2 (a^2 + 1) (b^2 + 1) CT + ((a^2 + 1) c2 + (b^2 + 1) c0) RT) +
          2 RT CT ((b T + a) (DT KT1 CT + DT KT CT1 - DT1 KT CT) - b DT CT KT) + 4 CT^4 (b T + a)
            (((a^2 - 1) c2 + (b^2 + 1) c0) (b T + a) - (1 / 2) a c1 RT1 + 2 a (c2 - c0) - b c1)

In[37]:= Expand[
          {Cancel[Z1] - RT CT^2 ZT /. {KT -> RT1 CT - RT CT1, KT1 -> 2 (b^2 + 1) CT - 2 c2 RT}} /.
           {RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
            CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T}]

Out[37]= {0}
```

Lemma 3.4

```
In[38]:= LTD := 4 ET

In[39]:= LTD1 := 3 ET1

In[40]:= LTC := 3 DT

In[41]:= LTC1 := 2 DT1

In[42]:= LTR := 4 (b T + a) CT

In[43]:= LTa := 2 c0 + c1 T

In[44]:= LTb := CT1

In[45]:= LTbTa := 2 CT
```

```
In[46]:= LTR1 := 4 b CT + 2 (b T + a) CT1

In[47]:= LTK := 3 RT1 DT - 2 RT DT1 + 2 CT (2 b CT - (b T + a) CT1)

In[48]:= LTK1 := 6 (b^2 + 1) DT - 2 (d2 + 3 T d3) RT + 4 (b c1 - 2 a c2) CT

In[49]:= LTc0 := 3 d0 + d1 T

In[50]:= LTc1 := 2 d1 + 2 d2 T

In[51]:= LTc2 := d2 + 3 d3 T

In[52]:= ST1 := DT1 RT + DT RT1 - 2 b CT^2 - 4 (b T + a) CT CT1

In[53]:= WT1 := b ST1 + CT1 KT + CT KT1

In[54]:= LTS := RT LTD + DT LTR - 2 LTbTa CT^2 - 4 (b T + a) CT LTC

In[55]:= LTW := ST LTb + b LTS + KT LTC + CT LTK

In[56]:= Expand[{KT (WT LTS - ST LTW) + 2 ST (WT ST1 - ST WT1) - 4 CT ZT /.
              {KT -> RT1 CT - RT CT1, KT1 -> 2 (b^2 + 1) CT - 2 c2 RT}} /.
          {RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
           CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T}]

Out[56]= {0}
```

Proposition 3.6.

(i)

```
In[57]:= Simplify[Expand[
           ZT / (c0 - c2) /. {KT → 2 T (c0 - c2), RT → T^2 + 1, CT → c0 + c2 T^2, a → 0, b → 0}]]
```

Out[57]= $2\,DT\,DT1\,T\left(1+T^2\right)^2-DT^2\left(1+6\,T^2+5\,T^4\right)-4\,(c0-c2)\,T^2\left(c0^3+3\,c0^2\,c2\,T^2+3\,c0\,c2^2\,T^4+c2^3\,T^6-ET\left(1+T^2\right)\right)$

In[58]:= $\text{Collect}\Big[\text{Expand}\Big[2\,DT\,DT1\,T\left(1+T^2\right)^2-DT^2\left(1+6\,T^2+5\,T^4\right)-4\,(c0-c2)\,T^2\left(c0^3+3\,c0^2\,c2\,T^2+3\,c0\,c2^2\,T^4+c2^3\,T^6-ET\left(1+T^2\right)\right)$ /.
{DT → d0 + d1 T + d2 T^2 + d3 T^3, DT1 → d1 + 2 d2 T + 3 d3 T^2,
ET → e0 + e1 T + e2 T^2 + e3 T^3 + e4 T^4}], T]

Out[58]= $-d0^2+\left(-4\,c0^4+4\,c0^3\,c2-6\,d0^2+d1^2+2\,d0\,d2+4\,c0\,e0-4\,c2\,e0\right)T^2+(-8\,d0\,d1+4\,d1\,d2+4\,d0\,d3+4\,c0\,e1-4\,c2\,e1)\,T^3+\left(-12\,c0^3\,c2+12\,c0^2\,c2^2-5\,d0^2-2\,d1^2-4\,d0\,d2+3\,d2^2+6\,d1\,d3+4\,c0\,e0-4\,c2\,e0+4\,c0\,e2-4\,c2\,e2\right)T^4+(-8\,d0\,d1+8\,d2\,d3+4\,c0\,e1-4\,c2\,e1+4\,c0\,e3-4\,c2\,e3)\,T^5+\left(-12\,c0^2\,c2^2+12\,c0\,c2^3-3\,d1^2-6\,d0\,d2+2\,d2^2+4\,d1\,d3+5\,d3^2+4\,c0\,e2-4\,c2\,e2+4\,c0\,e4-4\,c2\,e4\right)T^6+(-4\,d1\,d2-4\,d0\,d3+8\,d2\,d3+4\,c0\,e3-4\,c2\,e3)\,T^7+\left(-4\,c0\,c2^3+4\,c2^4-d2^2-2\,d1\,d3+6\,d3^2+4\,c0\,e4-4\,c2\,e4\right)T^8+d3^2\,T^{10}$

(iv)

```
In[59]:= a0 := s3 g7

In[60]:= a1 := - (s2 g7 + s3 g6)

In[61]:= a2 := s1 g7 + s2 g6 + s3 g5

In[62]:= a3 := - (g7 + s1 g6 + s2 g5 + s3 g4)
```

```
In[63]:= a4 := g6 + s1 g5 + s2 g4 + s3 g3

In[64]:= a5 := -(g5 + s1 g4 + s2 g3 + s3 g2)

In[65]:= a6 := g4 + s1 g3 + s2 g2 + s3 g1

In[66]:= a7 := -(g3 + s1 g2 + s2 g1 + s3)

In[67]:= a8 := g2 + s1 g1 + s2

In[68]:= a9 := -(g1 + s1)

In[69]:= Solve[{a1 == 0, a9 == 0, a5 - a3 - a7 == 0}, {s1, s2, s3}]

Out[69]= {{s1 -> -g1, s2 ->
     -(g6 (g1 g2 - g3 - g1 g4 + g5 + g1 g6 - g7)) / (-g1 g6 + g3 g6 - g5 g6 + g7 - g2 g7 + g4 g7),
   s3 -> (g7 (-g1 g2 + g3 + g1 g4 - g5 - g1 g6 + g7)) /
      (g1 g6 - g3 g6 + g5 g6 - g7 + g2 g7 - g4 g7)}}

In[70]:= G1 := g1 g6 - g3 g6 + g5 g6 - g7 + g2 g7 - g4 g7

In[71]:= G2 := g1 g2 - g3 - g1 g4 + g5 + g1 g6 - g7

In[72]:= s1 := -g1
```

In[73]:= $s2 := \frac{g6\ G2}{G1}$

In[74]:= $s3 := \frac{-g7\ G2}{G1}$

In[75]:= a0

Out[75]= $-\left((g1\ g2 - g3 - g1\ g4 + g5 + g1\ g6 - g7)\ g7^2\right) / (g1\ g6 - g3\ g6 + g5\ g6 - g7 + g2\ g7 - g4\ g7)$

In[76]:= Factor[a0 - a2 + a4 - a6 + a8 - 1]

Out[76]= $-\left((g1\ g6 - g7)\left(1 + g1^2 - 2\ g2 + g2^2 - 2\ g1\ g3 + g3^2 + 2\ g4 - 2\ g2\ g4 + g4^2 + 2\ g1\ g5 - 2\ g3\ g5 + g5^2 - 2\ g6 + 2\ g2\ g6 - 2\ g4\ g6 + g6^2 - 2\ g1\ g7 + 2\ g3\ g7 - 2\ g5\ g7 + g7^2\right)\right) / (g1\ g6 - g3\ g6 + g5\ g6 - g7 + g2\ g7 - g4\ g7)$

In[77]:= $\text{Simplify}\left[\left(1 + g1^2 - 2\ g2 + g2^2 - 2\ g1\ g3 + g3^2 + 2\ g4 - 2\ g2\ g4 + g4^2 + 2\ g1\ g5 - 2\ g3\ g5 + g5^2 - 2\ g6 + 2\ g2\ g6 - 2\ g4\ g6 + g6^2 - 2\ g1\ g7 + 2\ g3\ g7 - 2\ g5\ g7 + g7^2\right) - \left((g1 - g3 + g5 - g7)^2 + (1 - g2 + g4 - g6)^2\right)\right]$

Out[77]= 0

In[78]:= G3 := -(g1 g6 - g7)

In[79]:= $\text{Collect}\left[\left(\frac{S1}{3} + Y\right)^3 - S1\left(\frac{S1}{3} + Y\right)^2 + S2\left(\frac{S1}{3} + Y\right) - S3,\ Y\right]$

Out[79]= $-\frac{2\ S1^3}{27} + \frac{S1\ S2}{3} - S3 + \left(-\frac{S1^2}{3} + S2\right) Y + Y^3$

In[80]:=

$$\text{Factor}\left[4\left(-\left(-\frac{s1^2}{3}+s2\right)\Big/3\right)\text{^}3-\left(-\frac{2\,s1^3}{27}+\frac{s1\,s2}{3}-s3\right)\text{^}2\right]$$

Out[80]=

$$
\begin{aligned}
&-\Big((g1\,g2 - g3 - g1\,g4 + g5 + g1\,g6 - g7)\\
&\big(-g1^4\,g2\,g6^3 + 4\,g1^2\,g2^2\,g6^3 + g1^3\,g3\,g6^3 - 8\,g1\,g2\,g3\,g6^3 + g1^3\,g2\,g3\,g6^3 + 4\,g3^2\,g6^3 -\\
&g1^2\,g3^2\,g6^3 + g1^4\,g4\,g6^3 - 8\,g1^2\,g2\,g4\,g6^3 + 8\,g1\,g3\,g4\,g6^3 - g1^3\,g3\,g4\,g6^3 +\\
&4\,g1^2\,g4^2\,g6^3 - g1^3\,g5\,g6^3 + 8\,g1\,g2\,g5\,g6^3 - g1^3\,g2\,g5\,g6^3 - 8\,g3\,g5\,g6^3 +\\
&2\,g1^2\,g3\,g5\,g6^3 - 8\,g1\,g4\,g5\,g6^3 + g1^3\,g4\,g5\,g6^3 + 4\,g5^2\,g6^3 - g1^2\,g5^2\,g6^3 - g1^4\,g6^4 +\\
&8\,g1^2\,g2\,g6^4 - 8\,g1\,g3\,g6^4 + g1^3\,g3\,g6^4 - 8\,g1^2\,g4\,g6^4 + 8\,g1\,g5\,g6^4 - g1^3\,g5\,g6^4 +\\
&4\,g1^2\,g6^5 + 4\,g1^5\,g6^2\,g7 - 17\,g1^3\,g2\,g6^2\,g7 - g1^3\,g2^2\,g6^2\,g7 + 17\,g1^2\,g3\,g6^2\,g7 -\\
&8\,g1^4\,g3\,g6^2\,g7 + 19\,g1^2\,g2\,g3\,g6^2\,g7 - 18\,g1\,g3^2\,g6^2\,g7 + 4\,g1^3\,g3^2\,g6^2\,g7 +\\
&17\,g1^3\,g4\,g6^2\,g7 + 2\,g1^3\,g2\,g4\,g6^2\,g7 - 19\,g1^2\,g3\,g4\,g6^2\,g7 - g1^3\,g4^2\,g6^2\,g7 -\\
&17\,g1^2\,g5\,g6^2\,g7 + 8\,g1^4\,g5\,g6^2\,g7 - 19\,g1^2\,g2\,g5\,g6^2\,g7 + 36\,g1\,g3\,g5\,g6^2\,g7 -\\
&8\,g1^3\,g3\,g5\,g6^2\,g7 + 19\,g1^2\,g4\,g5\,g6^2\,g7 - 18\,g1\,g5^2\,g6^2\,g7 + 4\,g1^3\,g5^2\,g6^2\,g7 -\\
&16\,g1^3\,g6^3\,g7 - 8\,g1\,g2\,g6^3\,g7 - g1^3\,g2\,g6^3\,g7 + 8\,g3\,g6^3\,g7 + 17\,g1^2\,g3\,g6^3\,g7 +\\
&8\,g1\,g4\,g6^3\,g7 + g1^3\,g4\,g6^3\,g7 - 8\,g5\,g6^3\,g7 - 17\,g1^2\,g5\,g6^3\,g7 - 8\,g1\,g6^4\,g7 -\\
&8\,g1^4\,g6\,g7^2 + 45\,g1^2\,g2\,g6\,g7^2 + 8\,g1^4\,g2\,g6\,g7^2 - 18\,g1^2\,g2^2\,g6\,g7^2 -\\
&45\,g1\,g3\,g6\,g7^2 + 8\,g1^3\,g3\,g6\,g7^2 - 9\,g1\,g2\,g3\,g6\,g7^2 - 8\,g1^3\,g2\,g3\,g6\,g7^2 +\\
&27\,g3^2\,g6\,g7^2 - 45\,g1^2\,g4\,g6\,g7^2 - 8\,g1^4\,g4\,g6\,g7^2 + 36\,g1^2\,g2\,g4\,g6\,g7^2 +\\
&9\,g1\,g3\,g4\,g6\,g7^2 + 8\,g1^3\,g3\,g4\,g6\,g7^2 - 18\,g1^2\,g4^2\,g6\,g7^2 + 45\,g1\,g5\,g6\,g7^2 -\\
&8\,g1^3\,g5\,g6\,g7^2 + 9\,g1\,g2\,g5\,g6\,g7^2 + 8\,g1^3\,g2\,g5\,g6\,g7^2 - 54\,g3\,g5\,g6\,g7^2 -\\
&9\,g1\,g4\,g5\,g6\,g7^2 - 8\,g1^3\,g4\,g5\,g6\,g7^2 + 27\,g5^2\,g6\,g7^2 + 62\,g1^2\,g6^2\,g7^2 -\\
&17\,g1^2\,g2\,g6^2\,g7^2 - 45\,g1\,g3\,g6^2\,g7^2 + 17\,g1^2\,g4\,g6^2\,g7^2 + 45\,g1\,g5\,g6^2\,g7^2 +\\
&4\,g6^3\,g7^2 + 4\,g1^3\,g7^3 - 27\,g1\,g2\,g7^3 - 8\,g1^3\,g2\,g7^3 + 27\,g1\,g2^2\,g7^3 + 4\,g1^3\,g2^2\,g7^3 +\\
&27\,g3\,g7^3 - 27\,g2\,g3\,g7^3 + 27\,g1\,g4\,g7^3 + 8\,g1^3\,g4\,g7^3 - 54\,g1\,g2\,g4\,g7^3 -\\
&8\,g1^3\,g2\,g4\,g7^3 + 27\,g3\,g4\,g7^3 + 27\,g1\,g4^2\,g7^3 + 4\,g1^3\,g4^2\,g7^3 - 27\,g5\,g7^3 +\\
&27\,g2\,g5\,g7^3 - 27\,g4\,g5\,g7^3 - 72\,g1\,g6\,g7^3 + 45\,g1\,g2\,g6\,g7^3 + 27\,g3\,g6\,g7^3 -\\
&45\,g1\,g4\,g6\,g7^3 - 27\,g5\,g6\,g7^3 + 27\,g7^4 - 27\,g2\,g7^4 + 27\,g4\,g7^4\big)\Big)\Big/\\
&\big(27\,(g1\,g6 - g3\,g6 + g5\,g6 - g7 + g2\,g7 - g4\,g7)^3\big)
\end{aligned}
$$

In[81]:=
```
G4 := -g1^4 g2 g6^3 + 4 g1^2 g2^2 g6^3 + g1^3 g3 g6^3 - 8 g1 g2 g3 g6^3 + g1^3 g2 g3 g6^3 + 4 g3^2 g6^3 -
  g1^2 g3^2 g6^3 + g1^4 g4 g6^3 - 8 g1^2 g2 g4 g6^3 + 8 g1 g3 g4 g6^3 - g1^3 g3 g4 g6^3 + 4 g1^2 g4^2 g6^3 -
  g1^3 g5 g6^3 + 8 g1 g2 g5 g6^3 - g1^3 g2 g5 g6^3 - 8 g3 g5 g6^3 + 2 g1^2 g3 g5 g6^3 - 8 g1 g4 g5 g6^3 +
  g1^3 g4 g5 g6^3 + 4 g5^2 g6^3 - g1^2 g5^2 g6^3 - g1^4 g6^4 + 8 g1^2 g2 g6^4 - 8 g1 g3 g6^4 + g1^3 g3 g6^4 -
  8 g1^2 g4 g6^4 + 8 g1 g5 g6^4 - g1^3 g5 g6^4 + 4 g1^2 g6^5 + 4 g1^5 g6^2 g7 - 17 g1^3 g2 g6^2 g7 -
  g1^3 g2^2 g6^2 g7 + 17 g1^2 g3 g6^2 g7 - 8 g1^4 g3 g6^2 g7 + 19 g1^2 g2 g3 g6^2 g7 - 18 g1 g3^2 g6^2 g7 +
  4 g1^3 g3^2 g6^2 g7 + 17 g1^3 g4 g6^2 g7 + 2 g1^3 g2 g4 g6^2 g7 - 19 g1^2 g3 g4 g6^2 g7 -
  g1^3 g4^2 g6^2 g7 - 17 g1^2 g5 g6^2 g7 + 8 g1^4 g5 g6^2 g7 - 19 g1^2 g2 g5 g6^2 g7 + 36 g1 g3 g5 g6^2 g7 -
  8 g1^3 g3 g5 g6^2 g7 + 19 g1^2 g4 g5 g6^2 g7 - 18 g1 g5^2 g6^2 g7 + 4 g1^3 g5^2 g6^2 g7 -
  16 g1^3 g6^3 g7 - 8 g1 g2 g6^3 g7 - g1^3 g2 g6^3 g7 + 8 g3 g6^3 g7 + 17 g1^2 g3 g6^3 g7 +
  8 g1 g4 g6^3 g7 + g1^3 g4 g6^3 g7 - 8 g5 g6^3 g7 - 17 g1^2 g5 g6^3 g7 - 8 g1 g6^4 g7 - 8 g1^4 g6 g7^2 +
  45 g1^2 g2 g6 g7^2 + 8 g1^4 g2 g6 g7^2 - 18 g1^2 g2^2 g6 g7^2 - 45 g1 g3 g6 g7^2 + 8 g1^3 g3 g6 g7^2 -
  9 g1 g2 g3 g6 g7^2 - 8 g1^3 g2 g3 g6 g7^2 + 27 g3^2 g6 g7^2 - 45 g1^2 g4 g6 g7^2 - 8 g1^4 g4 g6 g7^2 +
  36 g1^2 g2 g4 g6 g7^2 + 9 g1 g3 g4 g6 g7^2 + 8 g1^3 g3 g4 g6 g7^2 - 18 g1^2 g4^2 g6 g7^2 +
  45 g1 g5 g6 g7^2 - 8 g1^3 g5 g6 g7^2 + 9 g1 g2 g5 g6 g7^2 + 8 g1^3 g2 g5 g6 g7^2 - 54 g3 g5 g6 g7^2 -
  9 g1 g4 g5 g6 g7^2 - 8 g1^3 g4 g5 g6 g7^2 + 27 g5^2 g6 g7^2 + 62 g1^2 g6^2 g7^2 - 17 g1^2 g2 g6^2 g7^2 -
  45 g1 g3 g6^2 g7^2 + 17 g1^2 g4 g6^2 g7^2 + 45 g1 g5 g6^2 g7^2 + 4 g6^3 g7^2 + 4 g1^3 g7^3 -
  27 g1 g2 g7^3 - 8 g1^3 g2 g7^3 + 27 g1 g2^2 g7^3 + 4 g1^3 g2^2 g7^3 + 27 g3 g7^3 - 27 g2 g3 g7^3 +
  27 g1 g4 g7^3 + 8 g1^3 g4 g7^3 - 54 g1 g2 g4 g7^3 - 8 g1^3 g2 g4 g7^3 + 27 g3 g4 g7^3 +
  27 g1 g4^2 g7^3 + 4 g1^3 g4^2 g7^3 - 27 g5 g7^3 + 27 g2 g5 g7^3 - 27 g4 g5 g7^3 - 72 g1 g6 g7^3 +
  45 g1 g2 g6 g7^3 + 27 g3 g6 g7^3 - 45 g1 g4 g6 g7^3 - 27 g5 g6 g7^3 + 27 g7^4 - 27 g2 g7^4 + 27 g4 g7^4
```

In[82]:=
```
g1 := SymmetricPolynomial[1, {x1, x2, x3, x4, x5, x6, x7}]
```

In[83]:=
```
g2 := SymmetricPolynomial[2, {x1, x2, x3, x4, x5, x6, x7}]
```

In[84]:=
```
g3 := SymmetricPolynomial[3, {x1, x2, x3, x4, x5, x6, x7}]
```

In[85]:=
```
g4 := SymmetricPolynomial[4, {x1, x2, x3, x4, x5, x6, x7}]
```

In[86]:=
```
g5 := SymmetricPolynomial[5, {x1, x2, x3, x4, x5, x6, x7}]
```

In[87]:=
```
g6 := SymmetricPolynomial[6, {x1, x2, x3, x4, x5, x6, x7}]
```

In[88]:=
```
g7 := SymmetricPolynomial[7, {x1, x2, x3, x4, x5, x6, x7}]
```

In[89]:=
```
G1 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[89]= $\frac{2\,436\,076}{5}$

In[90]:=
```
G2 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[90]= $\frac{664}{5}$

In[91]:=
```
G3 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[91]= $\frac{27\,382}{25}$

In[92]:=
```
G4 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[92]= $-\frac{4\,831\,113\,262\,058\,724\,096}{15\,625}$

```
In[93]:= a0 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[93]= $-\frac{382\,464}{609\,019}$

```
In[94]:= a1 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[94]= 0

```
In[95]:= a2 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[95]= $\frac{2\,505\,121\,816}{15\,225\,475}$

```
In[96]:= a3 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[96]= $-\frac{3\,662\,378\,874}{3\,045\,095}$

```
In[97]:= a4 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[97]= $\frac{29\,627\,935\,751}{15\,225\,475}$

```
In[98]:= a5 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[98]= $-\frac{5\,493\,568\,311}{6\,090\,190}$

```
In[99]:= a6 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[99]= $-\frac{14\,502\,009\,729}{60\,901\,900}$

```
In[100]:= a7 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[100]= $\frac{1\,831\,189\,437}{6\,090\,190}$

```
In[101]:= a8 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[101]= $-\frac{4\,181\,509\,819}{60\,901\,900}$

```
In[102]:= a9 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[102]= 0

```
In[103]:= a5 - a3 - a7 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[103]= 0

```
In[104]:= a0 - a2 + a4 - a6 + a8 - 1 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

Out[104]= $\frac{2\,374\,252\,147}{1\,218\,038}$

```
In[105]:= NSolve[(a0 + a1 X + a2 X^2 + a3 X^3 + a4 X^4 +
            a5 X^5 + a6 X^6 + a7 X^7 + a8 X^8 + a9 X^9 + X^{10} == 0 /.
          {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}), {X}]

Out[105]= {{X → -10.}, {X → -2.}, {X → -0.0519793}, {X → 0.1},
  {X → 0.13882}, {X → 1.}, {X → 1.81316}, {X → 2.}, {X → 3.}, {X → 4.}}
```

Lemma 3.8

In[106]:=
```
Solve[y == a x + b (x^2 + y^2), y]
```

Out[106]= $\left\{\left\{y \to \frac{1-\sqrt{1-4\,a\,b\,x-4\,b^2\,x^2}}{2\,b}\right\},\ \left\{y \to \frac{1+\sqrt{1-4\,a\,b\,x-4\,b^2\,x^2}}{2\,b}\right\}\right\}$

In[107]:= $\mathbf{Series}\left[\frac{1-\sqrt{1-4\,a\,b\,x-4\,b^2\,x^2}}{2\,b},\ \{x,\ 0,\ 4\}\right]$

Out[107]= $a\,x + \left(b + a^2\,b\right)\,x^2 + \left(2\,a\,b^2 + 2\,a^3\,b^2\right)\,x^3 + \left(b^3 + 6\,a^2\,b^3 + 5\,a^4\,b^3\right)\,x^4 + O[x]^5$

In[108]:=
```
P[x] := c0 x^2 + c2 y^2 + d0 x^3 + d1 x^2 y + d2 x y^2 +
    d3 y^3 + e0 x^4 + e1 x^3 y + e2 x^2 y^2 + e3 x y^3 + e4 y^4 /.
   {y → a x + (1 + a^2) b x^2 + 2 a b^2 (1 + a^2) x^3 + (1 + a^2) (1 + 5 a^2) b^3 x^4}
```

In[109]:=
```
Coefficient[P[x], x, 2]
```

Out[109]= $c0 + a^2\,c2$

In[110]:=
```
Coefficient[P[x], x, 3]
```

Out[110]= $2\,a\,b\,c2 + 2\,a^3\,b\,c2 + d0 + a\,d1 + a^2\,d2 + a^3\,d3$

In[111]:=
```
Coefficient[P[x], x, 4]
```

Out[111]= $b^2\,c2 + 6\,a^2\,b^2\,c2 + 5\,a^4\,b^2\,c2 + b\,d1 + a^2\,b\,d1 + 2\,a\,b\,d2 + 2\,a^3\,b\,d2 + 3\,a^2\,b\,d3 + 3\,a^4\,b\,d3 + e0 + a\,e1 + a^2\,e2 + a^3\,e3 + a^4\,e4$

The proof of Theorem 2.6.

To derive the explicit form of NT, we identify ZT with a function ZT[u], or ZT[T] where u and T are variables for differentiation in L_T and in T, respectively. (u is a fictional variable for L_T).

Therefore, LTZ : L_T[ZT] is given by

In[112]:=
```
ZTu[u] := ZT /. {KT → KT[u], RT → RT[u], ET → ET[u],
    CT → CT[u], DT → DT[u], DT1 → DT1[u], b → b[u], a → a[u], KT1 → KT1[u],
    c2 → c2[u], c0 → c0[u], CT1 → CT1[u], c1 → c1[u], RT1 → RT1[u]}
```

In[113]:=
```
LTZ := D[ZTu[u], u] /.
   {KT'[u] → LTK, KT[u] → KT, RT'[u] → LTR, RT[u] → RT, ET'[u] → LTE, ET[u] → ET,
    CT'[u] → LTC, CT[u] → CT, DT'[u] → LTD, DT[u] → DT, DT1'[u] → LTD1, DT1[u] → DT1,
    b'[u] → LTb, b[u] → b, a'[u] → LTa, a[u] → a, KT1'[u] → LTK1, KT1[u] → KT1,
    c2'[u] → LTc2, c2[u] → c2, c0'[u] → LTc0, c0[u] → c0, CT1'[u] → LTC1,
    CT1[u] → CT1, c1'[u] → LTc1, c1[u] → c1, RT1'[u] → LTR1, RT1[u] → RT1}
```

Z'[T] is given by the following ZT1 :

In[114]:=
```
ZTT[T] := ZT /. {KT → KT[T], RT → RT[T], ET → ET[T], CT → CT[T],
    DT → DT[T], DT1 → DT1[T], KT1 → KT1[T], CT1 → CT1[T], RT1 → RT1[T]}
```

In[115]:=
```
ZT1 := D[ZTT[T], T] /. {KT'[T] → KT1, KT[T] → KT, RT'[T] → RT1,
    RT[T] → RT, ET'[T] → ET1, ET[T] → ET, CT'[T] → CT1, CT[T] → CT,
    DT'[T] → DT1, DT[T] → DT, DT1'[T] → DT2, DT1[T] → DT1, KT1'[T] → KT2,
    KT1[T] → KT1, CT1'[T] → 2 c2, CT1[T] → CT1, RT1'[T] → RT2, RT1[T] → RT1}
```

The explicit form of N[T] is the following :

```
In[116]:= NT := -5 RT KT^2 ET1 (RT DT - 2 (b T + a) CT^2) + DT^3 RT B1 + 2 DT^2 DT1 RT^2 B2 -
     DT^2 RT^2 KT ((3 d3 T + d2) (5 RT - (b^2 + 1) T^2) + (d1 T + 3 d0) (b^2 + 1)) +
     DT^2 B3 + 2 (b T + a) DT DT1 RT CT B4 + 10 (b T + a) DT DT2 RT^2 KT CT^2 +
     DT B5 - 4 (b T + a) DT1 RT KT CT^3 (5 (b T + a) CT1 + 2 b CT) +
     4 (b T + a) CT^4 KT (3 d0 B6 + d1 B7 + d2 B8 - 3 d3 T B9) + 4 CT^4 B10

In[117]:= B1 := KT (42 (b^2 + 1) RT - 20 (a^2 + b^2 + 1)) -
     4 c2 RT (3 a b RT - 2 (a^2 + b^2 + 1 - a^2 b^2) T + 2 a b (a^2 + 1)) +
     2 (b^2 + 1) c1 RT (3 RT + 4 a b T + 4 a^2 + 4) - 4 (b^2 + 1) RT RT1 c0

In[118]:= B2 := -(4 (b^2 + 1) T + 5 a b) KT - ((b^2 + 1) T^2 + a^2 + 1) KT1 +
     4 (a^2 + 1) (b^2 + 1) CT - 2 ((a^2 + 1) c2 + (b^2 + 1) c0) RT

In[119]:= B3 := KT^2 (-4 a RT CT1 + 12 (b^2 + 1) (b T + a) T CT +
         8 a b (b T + a) CT + 12 b RT CT + 2 b RT (2 c0 + c1 T)) +
     KT KT1 (4 a b T (b T + a) CT - 12 (b T + a) RT CT + b T RT (2 c0 + c1 T) + a T RT CT1) +
     KT (-4 a (b^2 + 6) RT CT^2 - (18 (b^2 + 1) T (b T + a) + 16 a b (b T + a) - 4 b (a^2 + 1))
          RT CT CT1 + 8 (a^2 + 1) (b^2 + 1) (b T + a) CT^2 -
         8 (b T + a) ((a^2 + 1) c2 + (b^2 + 1) c0) RT CT + 4 a b T (b c1 - 2 a c2) RT CT +
         4 a (b^2 + 1) (2 c0 + c1 T) RT CT - 2 a c2 (2 c0 + c1 T) RT^2 - 2 b c0 CT1 RT^2 -
         12 (b^2 + 1) (b T + a) T RT1 CT^2 - 8 a b (b T + a) RT1 CT^2 - 4 (b T + a) RT RT1 CT CT1 +
         4 b RT RT1 CT^2 - 4 (b T + a) RT^2 CT1^2 - 8 c2 (b T + a) RT^2 CT + 4 b RT^2 CT CT1) -
     2 (b T + a) KT1 CT (2 a b T RT1 CT + 5 RT1 RT CT + 6 RT^2 CT1) -
     4 (b T + a) KT2 RT CT^2 (RT + a b T) -
     8 (a^2 + 1) (b^2 + 1) (b T + a) RT1 CT^3 +
     8 (b T + a) ((a^2 + 1) c2 + (b^2 + 1) c0) RT RT1 CT^2 -
     8 (a^2 + 1) (b^2 + 1) (b T + a) RT CT^2 CT1

In[120]:= B4 := 2 (3 (b^2 + 1) T + 5 a b) KT CT + 5 RT KT CT1 + 2 ((b^2 + 1) T^2 + a^2 + 1) KT1 CT -
     8 (a^2 + 1) (b^2 + 1) CT^2 + 4 RT CT ((a^2 + 1) c2 + (b^2 + 1) c0)

In[121]:= B5 := 9 CT^2 KT^3 + KT^2 CT^2 (6 RT1 CT + 4 RT CT1 + 8 b (b T + a) CT - 8 (b T + a)^2 CT1) -
     8 (b T + a)^2 KT KT1 CT^3 +
     KT CT^2 (-8 (b T + a) (b c1 - 2 a c2) RT CT - 8 b (b T + a) RT CT1 CT + 8 b^2 RT CT^2 -
         4 b (b T + a) RT CT CT1 - 48 (b T + a)^2 CT ((a^2 - 1) c2 + (b^2 + 1) c0) +
         24 a c1 (b T + a) RT1 CT - 48 (b T + a) CT (2 a (c2 - c0) - b c1) +
         12 (b T + a)^2 RT CT1^2 - 8 b (b T + a) RT1 CT^2 + 8 (b T + a)^2 RT1 CT CT1 +
         16 c2 (b T + a)^2 RT CT - 8 b (b T + a) RT CT CT1) +
     8 (b T + a)^2 KT1 CT^3 (RT1 CT + 3 RT CT1) + 8 (b T + a)^2 RT CT^4 KT2 -
     16 b (b T + a) RT CT^4 ((a^2 - 1) c2 + (b^2 + 1) c0) -
     32 (b T + a)^2 RT CT^3 CT1 ((a^2 - 1) c2 + (b^2 + 1) c0) +
     16 a (b T + a) RT RT1 c1 CT^3 CT1 + 4 a b RT RT1 c1 CT^4 +
     8 a (b^2 + 1) (b T + a) RT c1 CT^4 - 8 b RT CT^4 (2 a (c2 - c0) - b c1) -
     32 (b T + a) RT CT^3 CT1 (2 a (c2 - c0) - b c1)

In[122]:= B6 := -(b^2 + 1) (b T + a) + 2 a

In[123]:= B7 := -(b^2 + 1) T (b T + a) + a RT1 + 2 (a T + b)

In[124]:= B8 := -(a^2 - 3) (b T + a) + a T RT1 - 4 a - 4 (b T + a) RT

In[125]:= B9 := (a^2 - 1) (b T + a) + 2 a + 4 (b T + a) RT
```

In[126]:=
```
B10 := KT^2 (2 b CT - 4 (b T + a) CT1) - 2 (b T + a) CT KT KT1 +
  KT (a RT1 c1 CT - 4 (b T + a) CT ((a^2 - 1) c2 + (b^2 + 1) c0) - 4 a (c2 - c0) CT +
     2 b c1 CT - 2 a (b T + a) ^2 c2 (2 c0 + c1 T) - 2 b (b T + a) ^2 c0 CT1 +
     2 a b (b T + a) c1 CT + a (b T + a) ^2 c1 CT1 + (1 / 2) (b T + a) RT1 c1 (2 c0 + c1 T) -
     2 (b T + a) (c2 - c0) (2 c0 + c1 T) + (b T + a) c1 CT1) +
  8 b (b T + a) ^2 ((a^2 - 1) c2 + (b^2 + 1) c0) CT^2 +
  16 (b T + a) ^3 CT CT1 ((a^2 - 1) c2 + (b^2 + 1) c0) -
  8 a (b T + a) ^2 RT1 c1 CT CT1 - 2 a b (b T + a) RT1 c1 CT^2 -
  4 a (b^2 + 1) (b T + a) ^2 c1 CT^2 + 4 b (b T + a) CT^2 (2 a (c2 - c0) - b c1) +
  16 (b T + a) ^2 CT CT1 (2 a (c2 - c0) - b c1)
```

On the other hand, the definition of N[T] is given by the following :

In[127]:=
```
NTdef := -KT (LTZ - KT^2 RT LTE) - 2 ST ZT1
```

In[128]:=
```
Expand[{NT - NTdef /. {KT -> RT1 CT - RT CT1,
     KT1 -> 2 (b^2 + 1) CT - 2 c2 RT, KT2 -> 2 (b^2 + 1) CT1 - 2 c2 RT1}} /.
  {RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
   RT2 -> 2 (b^2 + 1), CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T, CT2 -> 2 c2,
   DT -> d0 + d1 T + d2 T^2 + d3 T^3, DT1 -> d1 + 2 T d2 + 3 d3 T^2, DT2 -> 2 d2 + 6 d3 T}]
```

Out[128]= {0}

In[129]:=
```
Simplify[Coefficient[{NT /. {KT -> RT1 CT - RT CT1,
       KT1 -> 2 (b^2 + 1) CT - 2 c2 RT, KT2 -> 2 (b^2 + 1) CT1 - 2 c2 RT1}} /.
    {RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
     RT2 -> 2 (b^2 + 1), CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T,
     CT2 -> 2 c2, DT -> d0 + d1 T + d2 T^2 + d3 T^3, DT1 -> d1 + 2 T d2 + 3 d3 T^2,
     DT2 -> 2 d2 + 6 d3 T, ET1 → e1 + 2 e2 T + 3 e3 T^2 + 4 e4 T^3}, T, 14]]
```

Out[129]=

$$\{-20\,(-2\,b\,c2^2+d3+b^2\,d3)\,\Big(2\,b^5\,c2\,(-2\,c0\,c2+a\,d2)\,d3+4\,b^3\,c2\,(-2\,c0\,c2+c2^2+a\,d2)\,d3+2\,b\,c2\,(-2\,c0\,c2+2\,c2^2+a\,d2)\,d3+b^6\,c0\,d3^2+(c0-(1+a^2)\,c2)\,d3^2+(1+b^2)^2\,c1^2\,(-c2^3-2\,b\,c2\,d3+e4+b^2\,e4)-(1+b^2)\,c1\,\big(b^4\,d2\,d3+2\,b^2\,(-3\,a\,c2^2+d2)\,d3+(-2\,a\,c2^2+d2)\,d3-2\,b\,(c2^2\,d2+a\,d3^2-2\,a\,c2\,e4)-2\,b^3\,(c2^2\,d2+a\,d3^2-2\,a\,c2\,e4)\big)+b^2\,\big(c0\,(4\,c2^4+3\,d3^2)-2\,c2\,(2\,c2^4+2\,a\,c2^2\,d2+d3^2+3\,a^2\,d3^2-2\,a^2\,c2\,e4)\big)+b^4\,\big(c0\,(4\,c2^4+3\,d3^2)-c2\,(4\,a\,c2^2\,d2+d3^2+a^2\,(5\,d3^2-4\,c2\,e4))\big)\Big)\}$$